

Bitopological Harmonious Labeling Of Some Acyclic Graphs By Using Python Program

S. Chandrakala¹, G. Siva Prijith², M. Subbulakshmi³

¹Associate Professor, PG & Research Department of Mathematics,

T.D.M.N.S College, T.Kallikulam. Email ID : chandrakalakamaraj@gmail.com

²Research Scholar, Reg. No. 19222052092004, G. Venkataswamy Naidu College, Kovilpatti.

Affiliated to Manonmaniam Sundaranar University, Tirunelveli.

³Associate Professor, PG and Research Department of Mathematics, G. Venkataswamy Naidu College, Kovilpatti.

Abstract

In this paper, we define a new type of labeling called bitopological harmonious labeling in such a way that for a graph $G = (V(G), E(G))$ with n vertices, bitopological harmonious labeling is an injective function $f: V(G) \rightarrow 2^X$, where X is any non – empty set such that $|X| = m$, $m < n$ and $\{f(V(G))\}$ forms a topology on X , that induces an injective function $f^*: E(G) \rightarrow 2^{X^*}$, defined by $f^*(uv) = f(u) \cap f(v)$ for every $uv \in E(G)$ such that $\{f^*(E(G))\}$ forms a topology on X^* where $X^* = X/\{1,2, \dots, m\}$. A graph that admits bitopological harmonious labeling is called a bitopological harmonious graph. In this paper, we discuss bitopological harmonious labeling of some acyclic graphs and to generate these labels by using python program code.

Keywords: Bitopological harmonious graph, star graph, path, shrub graph.

I INTRODUCTION

In this paper we consider only simple, finite and undirected graphs. The graph G has a vertex set $V = V(G)$ and edge set $E = E(G)$. For notations and terminology we refer to Bondy and Murthy[2]. Acharya [3] established another link between graph theory and point set topology. Selestin Lina S and Asha S defined bitopological star labeling for a graph $G = (V, E)$ as X be any non-empty set if there exists an injective function $f: V(G) \rightarrow 2^X$ which induces the function $f^*: E(G) \rightarrow 2^X$ as $f^*(v_1v_2) = [f(v_1) \cup f(v_2)]^c$ for every $v_1, v_2 \in V(G)$, if $\{f(V(G))\}$ and $\{f^*(E(G))\} \cup X$ are topologies on X then G is said to be bitopological star graph. In this paper, we defined bitopological harmonious labeling and proved star graph, path graph, comb graph and shrub graph are bitopological harmonious graph. Also we discussed to generate the label of these graphs by using python program code.

II BASIC DEFINITIONS

Definition 2.1

Bitopological harmonious labeling of a graph $G = (V(G), E(G))$ with n vertices is an injective function $f: V(G) \rightarrow 2^X$, where X is any non – empty set such that $|X| = m$, $m < n$ and $\{f(V(G))\}$ forms a topology on X , that induces an injective function $f^*: E(G) \rightarrow 2^{X^*}$, defined by $f^*(uv) = f(u) \cap f(v)$ for every $uv \in E(G)$ such that $\{f^*(E(G))\}$ forms a topology on X^* where $X^* = X/\{1,2, \dots, m\}$. A graph that admits bitopological harmonious labeling is called a bitopological harmonious graph.

Definition 2.2

A graph is said be acyclic if it has no cycles.

Definition 2.3

A graph obtained from a path P_n by attaching pendent edges to every internal vertices of the path is called hurdle graph. Hurdle graph with $n - 2$ hurdle is denoted by Hd_n .

Definition 2.4

Shrub graph is a graph obtained by connecting a vertex to the centre vertex of each n number of star graph K_{1,m_i} . It is denoted by $S(m_1, m_2, \dots, m_n)$.

Definition 2.5

The corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 and joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2 .

III MAIN RESULTS

Theorem 3.1

The path $P_n, n > 1$ is a bitopological harmonious graph.

Proof:

Let $V(P_n) = \{u_1, u_2, \dots, u_n\}$.

Let $E(P_n) = \{u_i u_{i+1} / 1 \leq i \leq n - 1\}$.

$|V(G)| = n, |E(G)| = n - 1$.

Let $X = \{1, 2, \dots, |V(P_n)| - 1\}$.

Define the function $f: V(P_n) \rightarrow 2^X$ as follows:

$f(u_1) = \phi$;

$f(u_i) = \{1, 2, \dots, i - 1\}$ for $2 \leq i \leq n$.

Here all the vertex labels are distinct and they form a topology on X .

Then the induced function $f^*: E(P_n) \rightarrow 2^{X^*}$ is given as follows:

$f^*(uv) = f(u) \cap f(v)$ for all $uv \in E(P_n)$.

Here $f^*(u_i u_{i+1}) = f(u_i)$ for $1 \leq i \leq n - 1$.

Since f is 1-1 and so f^* . Also $\{f^*(E(P_n))\}$ forms a topology on X^* .

Hence f is a bitopological harmonious labeling and P_n is a bitopological harmonious graph.

To generate the bitopological harmonious labeling of P_n using python programming code:

Program:

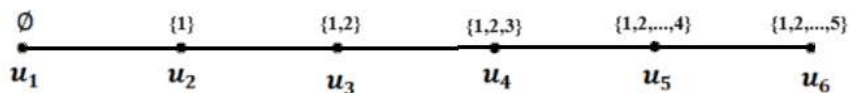
```
import sys
n=int(input("Enter the value of Pn\n"))
if n==2:
print("f(u1)=empty set,f(u2)={1}")
elif n==3:
print("f(u1)=empty set,f(u2)={1},f(u3)={1,2}")
else:
for i in range(1):
print("f(u1)=empty set,f(u2)={1},f(u3)={1,2}")
for i in range (4,n+1):
print("f(u",i,")={1,...,",i-1,}")")
```

Output:

```

⇒ Enter the value of Pn
10
f(u1)=empty set, f(u2)={1}, f(u3)={1,2}
f(u 4 )={1,..., 3 }
f(u 5 )={1,..., 4 }
f(u 6 )={1,..., 5 }
f(u 7 )={1,..., 6 }
f(u 8 )={1,..., 7 }
f(u 9 )={1,..., 8 }
f(u 10 )={1,..., 9 }

```

Example 3.2**Fig 3.1 Bitopological harmonious labeling of P_6** **Theorem 3.3**

Star graph $K_{1,n}$, $n > 1$ is a bitopological harmonious graph.

Proof:

Let $G = K_{1,n}$.

Let $V(G) = \{u_0, u_1, \dots, u_n\}$ be the vertex set of G and u_0 be the centre vertex.

Let $E(G) = \{u_0u_i / 1 \leq i \leq n\}$ be the edge set of G .

Let $X = \{1, 2, \dots, |V(G)| - 1\}$.

Define the function $f: V(G) \rightarrow 2^X$ as follows:

$$f(u_1) = \phi;$$

$$f(u_i) = \{1, 2, \dots, i - 1\} \text{ for } 2 \leq i \leq n;$$

$$f(u_0) = \{1, 2, \dots, n\}.$$

Here all the vertex labels are distinct and they form a topology on X .

Then the induced function $f^*: E(G) \rightarrow 2^{X^*}$ is given as follows:

$$f^*(uv) = f(u) \cap f(v) \text{ for all } uv \in E(G).$$

Here $f^*(u_0u_i) = f(u_i)$ for $1 \leq i \leq n$.

Since f is 1-1 and so f^* . Also $\{f^*(E(G))\}$ forms a topology on X^* .

Hence f is a bitopological harmonious labeling and G is a bitopological harmonious graph.

To generate the bitopological harmonious labeling of G using python programming code:

Program:

```

import sys
n=int(input("Enter the value of n \n"))
if n==1:
    print("f(u0)={1} f(u1)=empty set")

```

elif n==2:

```
print("f(u0)={1,2}, f(u1)=empty set, f(u2)={1}")
```

else:

```
for i in range(1):
```

```
    print("f(u0)={1,...,n}, f(u1)=empty set, f(u2)={1}")
```

```
for i in range(3,n+1):
```

```
    print("f(u",i,")={1,...,i-1}")
```

Output:

```

Enter the value of n
6
f(u0)={1,..., 6 }, f(u1)=empty set, f(u2)={1}
f(u 3 )={1,..., 2 }
f(u 4 )={1,..., 3 }
f(u 5 )={1,..., 4 }
f(u 6 )={1,..., 5 }

```

Example 3.4

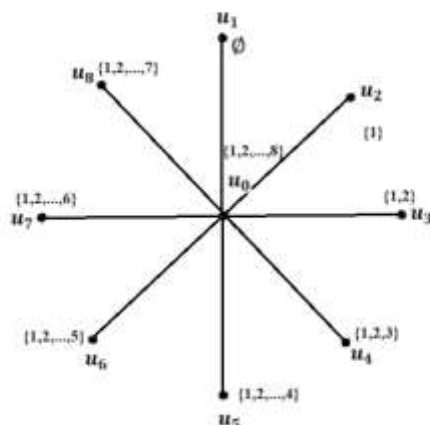


Fig 3.2 Bitopological harmonious labeling of $K_{1,8}$

Theorem 3.5

For any $n \in \mathbb{N}$ the comb graph $P_n \odot K_1$, is a bitopological harmonious graph.

Proof:

Let $G = P_n \odot K_1$ and let $V(G) = \{u_1, u_2, \dots, u_n, v_1, \dots, v_n\}$ be the vertex set of G .

Let $E(G) = \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i v_i / 1 \leq i \leq n\}$.

Then $|V(G)| = 2n$, $|E(G)| = 2n - 1$.

Let $X = \{1, 2, \dots, 2n - 1\}$.

Define the function $f: V(G) \rightarrow 2^X$ as follows:

$f(u_1) = \emptyset$;

$f(v_i) = \{1, 2, \dots, 2(i-1) + 1\}$ for $1 \leq i \leq n$;

$f(u_i) = \{1, 2, \dots, 2(i-1)\}$ for $1 \leq i \leq n$.

Here all the vertex labels are distinct and they form a topology on X .

Then the induced function $f^*: E(G) \rightarrow 2^{X^*}$ is given as follows:

$$f^*(uv) = f(u) \cap f(v) \quad \text{for all } uv \in E(G).$$

Here $f^*(v_i v_{i+1}) = f(v_i)$ for $1 \leq i \leq n-1$;

$$f^*(u_i v_i) = f(u_i) \quad \text{for } 1 \leq i \leq n-1.$$

Since f is 1-1 and so f^* . Also $\{f^*(E(G))\}$ forms a topology on X^* .

Hence f is a bitopological harmonious labeling and G is a bitopological harmonious graph.

To generate the bitopological harmonious labeling of comb graph $P_n \odot K_1$ using python programming code:

```
import sys
n=int(input("Enter the value of n\n"))
if n==1:
    print("f(u1)=empty set,f(v1)={1}")
elif n==2:
    print("f(u1)=empty set,f(v1)={1},\nf(u2)={1,2},f(v2)={1,2,3}")
else:
    for i in range(1):
        print("f(u1)=empty set,f(v1)={1},\nf(u2)={1,2},f(v2)={1,2,3}")
    for i in range (3,n+1):
        print("f(u",i,")={1,...,(i-1)*2},f(v",i,")={1,2,...,(i-1)*2+1}")
```

Output:

```
Enter the value of n
5
f(u1)=empty set, f(v1)={1},
f(u2)={1,2}, f(v2)={1,2,3}
f(u 3 )={1, ..., 4 }, f(v 3 )={1,2, ..., 5 }
f(u 4 )={1, ..., 6 }, f(v 4 )={1,2, ..., 7 }
f(u 5 )={1, ..., 8 }, f(v 5 )={1,2, ..., 9 }
```

Example 3.6

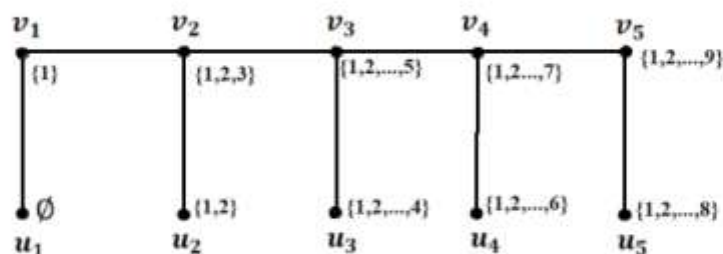


Fig 3.3 Bitopological harmonious labeling of $P_5 \odot K_1$

Theorem 3.7

For any $n > 2$, the hurdle graph Hd_n , is a bitopological harmonious graph.

Proof:

Let $G = Hd_n$.

Let $V(G) = \{u_1, u_2, \dots, u_n, v_1, \dots, v_{n-2}\}$.

Let $E(G) = \{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i v_{i-1} / 2 \leq i \leq n-1\}$.

Then $|V(G)| = 2n - 2$, $|E(G)| = 2n - 3$.

Let $X = \{1, 2, \dots, 2n - 3\}$.

Define the function $f: V(G) \rightarrow 2^X$ as follows:

$f(v_1) = \phi$;

$f(v_i) = \{1, 2, \dots, i - 1\}$ for $2 \leq i \leq n - 1$;

$f(u_i) = \{1, 2, \dots, n - 3 + i\}$ for $1 \leq i \leq n$.

Here all the vertex labels are distinct and they form a topology on X .

Then the induced function $f^*: E(G) \rightarrow 2^{X^*}$ is given as follows:

$f^*(uv) = f(u) \cap f(v)$ for all $uv \in E(G)$.

Here $f^*(u_i u_{i+1}) = f(u_i)$ for $1 \leq i \leq n - 1$;

$f^*(u_i v_{i-1}) = f(v_i)$ for $2 \leq i \leq n - 1$.

Since f is 1-1 and so f^* . Also $\{f^*(E(G))\}$ forms a topology on X^* .

Hence f is a bitopological harmonious labeling and G is a bitopological harmonious graph.

To generate the bitopological harmonious labeling of hurdle graph Hd_n using python programming code:

Program:

```
import sys
n=int(input("Enter the value of n\n"))
print("f(v1)=empty set")
if n==3:
    print("f(u1)={1},\nf(u2)={1,2},f(u3)={1,2,3}")
else:
    for i in range(2,n-1):
        print("f(v",i,")={1,...,",i-1,}")
    for i in range (1,n+1):
        print("f(u",i,")={1,...,",n-3+i,}")
```

Output:

```

Enter the value of n
8
f(v1)=empty set
f(v 2 )={1,..., 1 }
f(v 3 )={1,..., 2 }
f(v 4 )={1,..., 3 }
f(v 5 )={1,..., 4 }
f(v 6 )={1,..., 5 }
f(u 1 )={1,..., 6 }
f(u 2 )={1,..., 7 }
f(u 3 )={1,..., 8 }
f(u 4 )={1,..., 9 }
f(u 5 )={1,..., 10 }
f(u 6 )={1,..., 11 }
f(u 7 )={1,..., 12 }
f(u 8 )={1,..., 13 }

```

Example 3.8

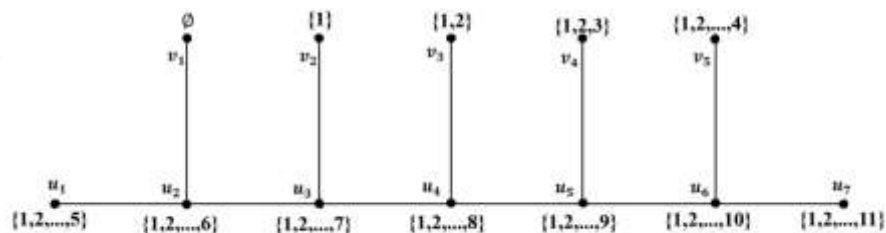


Fig 3.4 Bitopological harmonious labeling of Hd_7

Theorem 3.9

For any $n \in \mathbb{N}$, two pendent edges attached to each vertices of P_n is a bitopological harmonious graph.

Proof:

Let G be the graph obtained by attaching two pendent edges to each vertices of P_n .

Let $V(G) = \{u_1, u_2, \dots, u_n, v_1, \dots, v_n, w_1, \dots, w_n\}$ be the vertex set of G where u_i and w_i be the pendent vertices attached to v_i .

Let $E(G) = \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_i u_i / 1 \leq i \leq n\} \cup \{v_i w_i / 1 \leq i \leq n\}$.

Then $|V(G)| = 3n$, $|E(G)| = 3n - 1$.

Let $X = \{1, 2, \dots, 3n - 1\}$.

Define the function $f: V(G) \rightarrow 2^X$ as follows:

$$f(u_1) = \phi;$$

$$f(u_i) = \{1, 2, \dots, 3(i-1)\} \text{ for } 2 \leq i \leq n;$$

$$f(w_i) = \{1, 2, \dots, 3(i-1) + 1\} \text{ for } 1 \leq i \leq n;$$

$$f(v_i) = \{1, 2, \dots, 3(i-1) + 2\} \text{ for } 1 \leq i \leq n.$$

Here all the vertex labels are distinct and they form a topology on X .

Then the induced function $f^*: E(G) \rightarrow 2^X$ is given as follows:

$$f^*(uv) = f(u) \cap f(v) \text{ for all } uv \in E(G).$$

$$\text{Here } f^*(v_i v_{i+1}) = f(v_i) \text{ for } 1 \leq i \leq n-1;$$

$$f^*(u_i v_i) = f(u_i) \text{ for } 1 \leq i \leq n;$$

$$f^*(v_i w_i) = f(w_i) \text{ for } 1 \leq i \leq n.$$

Since f is 1-1 and so f^* . Also $\{f^*(E(G))\}$ forms a topology on X^* .

Hence f is a bitopological harmonious labeling and G is a bitopological harmonious graph.

To generate the bitopological harmonious labeling of G using python programming code:

Program:

```
import sys
n=int(input("Enter the value of n\n"))
if n==1:
    print("f(u1)=empty set,f(w1)={1},f(v1)={1,2}")
elif n==2:
    print("f(u1)=empty set,f(w1)={1},f(v1)={1,2},\nf(u2)={1,2,3},f(w2)={1,2,...,4},f(v2)={1,2,...,5}")
else:
    for i in range(1):
        print("f(u1)=empty set,f(w1)={1},f(v1)={1,2},\nf(u2)={1,2,3},f(w2)={1,2,...,4},f(v2)={1,2,...,5}")
    for i in range (3,n+1):
        print("f(u",i,")={1,...,(i-1)*3},f(w",i,")={1,...,(i-1)*3+1},f(v",i,")={1,2,...,(i-1)*3+2}")
```

Output:

```
Enter the value of n
5
f(u1)=empty set,f(w1)={1},f(v1)={1,2},
f(u2)={1,2,3},f(w2)={1,2,...,4},f(v2)={1,2,...,5}
f(u 3 )={1,..., 6 },f(w 3 )={1,..., 7 },f(v 3 )={1,2,..., 8 }
f(u 4 )={1,..., 9 },f(w 4 )={1,..., 10 },f(v 4 )={1,2,..., 11 }
f(u 5 )={1,..., 12 },f(w 5 )={1,..., 13 },f(v 5 )={1,2,..., 14 }
```

Example 3.10

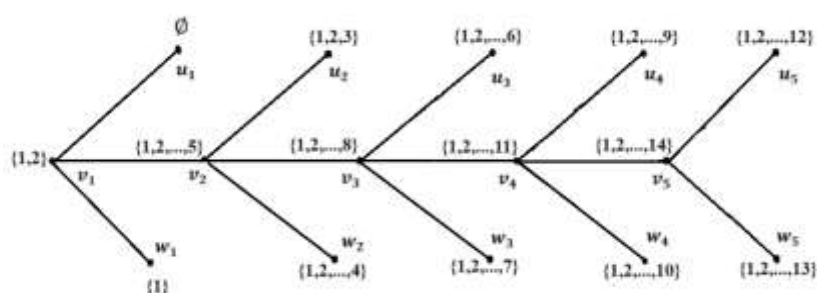


Fig 3.5 Bitopological harmonious labeling of two pendent edges attached to each vertices of P_5

Theorem 3.11

For any $n, m \in \mathbb{N}$ the graph $P_n \odot K_{1,m}$, is a bitopological harmonious graph.

Proof:

Let $G = P_n \odot K_{1,m}$ and let $V(G) = \{v_j^{(i)} / 1 \leq i \leq n, 0 \leq j \leq m\}$ be the vertex set of G .

$$\text{Let } E(G) = \{v_0^{(i)}v_0^{(i+1)} / 1 \leq i \leq n-1\} \cup \{v_0^{(i)}v_j^{(i)} / 1 \leq i \leq n, 1 \leq j \leq m\}.$$

$$\text{Then } |V(G)| = n(m+1) \text{ and } |E(G)| = n(m+1) - 1.$$

$$\text{Let } X = \{1, 2, \dots, n(m+1) - 1\}.$$

Define the function $f: V(G) \rightarrow 2^X$ as follows:

$$f(v_1^{(1)}) = \phi;$$

$$f(v_0^{(i)}) = \{1, 2, \dots, (m+1)i - 1\} \text{ for } 1 \leq i \leq n;$$

$$f(v_j^{(i)}) = \{1, 2, \dots, (m+1)(i-1) + (j-1)\} \text{ for } 1 \leq j \leq m;$$

$$f(u_i) = \{1, 2, \dots, 2(i-1)\} \text{ for } 1 \leq i \leq n.$$

Here all the vertex labels are distinct and they form a topology on X .

Then the induced function $f^*: E(G) \rightarrow 2^{X^*}$ is given as follows:

$$f^*(uv) = f(u) \cap f(v) \text{ for all } uv \in E(G).$$

$$\text{Here } f^*(v_0^{(i)}v_0^{(i+1)}) = f(v_0^{(i)}) \text{ for } 1 \leq i \leq n-1;$$

$$f^*(v_0^{(i)}v_j^{(i)}) = f(v_j^{(i)}) \text{ for } 1 \leq i \leq n, 1 \leq j \leq m.$$

Since f is 1-1 and so f^* . Also $\{f^*(E(G))\}$ forms a topology on X^* .

Hence f is a bitopological harmonious labeling and G is a bitopological harmonious graph.

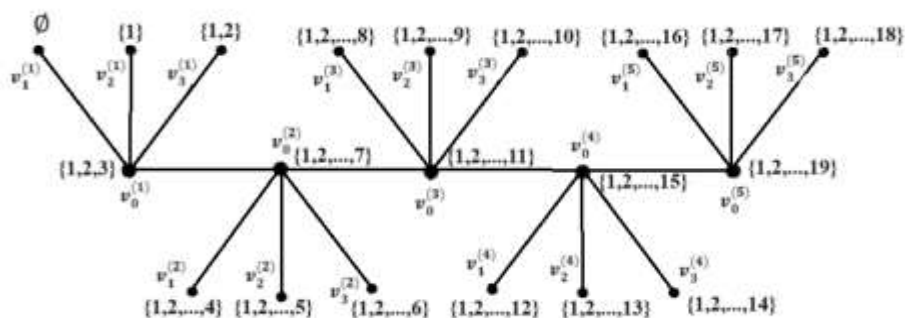
To generate the bitopological harmonious labeling of $P_n \odot K_{1,m}$ using python programming code:

Program:

```
import sys
n=int(input("Enter the value of n\n"))
m=int(input("Enter the value of m\n"))
for i in range(1,n+1):
    print("f(v^",i,"_0)={1,2,...,", (m+1)*i-1, "}")
    for j in range(1,m+1):
        print("f(v^",i,"_",j,")={1,2,...,", (m+1)*(i-1)+j-1, "}")
```

Output:

```
Enter the value of n
3
Enter the value of m
4
f(v^ 1 _0)={1,2,..., 4 }
f(v^ 1 _1 )={1,2,..., 0 }
f(v^ 1 _2 )={1,2,..., 1 }
f(v^ 1 _3 )={1,2,..., 2 }
f(v^ 1 _4 )={1,2,..., 3 }
f(v^ 2 _0)={1,2,..., 9 }
f(v^ 2 _1 )={1,2,..., 5 }
f(v^ 2 _2 )={1,2,..., 6 }
f(v^ 2 _3 )={1,2,..., 7 }
f(v^ 2 _4 )={1,2,..., 8 }
f(v^ 3 _0)={1,2,..., 14 }
f(v^ 3 _1 )={1,2,..., 10 }
f(v^ 3 _2 )={1,2,..., 11 }
f(v^ 3 _3 )={1,2,..., 12 }
f(v^ 3 _4 )={1,2,..., 13 }
```

Example 3.12**Fig 3.6 Bitopological harmonious labeling of $P_5 \odot K_{1,3}$** **Theorem 3.13**

Shrub graph $S(m_1, m_2, \dots, m_n), n > 1, m_1, m_2, \dots, m_n \in N$ is a bitopological harmonious graph.

Proof:

Let $G = S(m_1, m_2, \dots, m_n)$ be a Shrub graph.

Let $V(G) = \{c, v_i, v_j^{(i)} / 1 \leq i \leq n, 1 \leq j \leq m_i\}$ and c be the centre vertex of G .

Let $E(G) = \{cv_i / 1 \leq i \leq n\} \cup \{v_i v_j^{(i)} / 1 \leq i \leq n, 1 \leq j \leq m_i\}$.

Then $|V(G)| = m_1 + m_2 + \dots + m_n + n + 1$ and $|E(G)| = m_1 + m_2 + \dots + m_n + n$.

Let $X = \{1, 2, \dots, m_1 + m_2 + \dots + m_n + n\}$.

Define the function $f: V(G) \rightarrow 2^X$ as follows:

$$f(v_1^{(1)}) = \phi;$$

$$f(v_j^{(1)}) = \{1, 2, \dots, j-1\} \text{ for } j = 2, 3, \dots, m_1;$$

$$f(v_j^{(i)}) = \{1, 2, \dots, m_1 + m_2 + \dots + m_{i-1} + (i-2) + j\} \text{ for } 2 \leq i \leq n \text{ and } 1 \leq j \leq m_i;$$

$$f(v_i) = \{1, 2, \dots, m_1 + m_2 + \dots + m_i + (i-1)\} \text{ for } 1 \leq i \leq n;$$

$$f(c) = \{1, 2, \dots, m_1 + m_2 + \dots + m_n + n\}.$$

Here all the vertex labels are distinct and they form a topology on X .

Then the induced function $f^*: E(G) \rightarrow 2^{X^*}$ is given as follows:

$$f^*(uv) = f(u) \cap f(v) \text{ for all } uv \in E(G).$$

Here $f^*(cv_i) = f(v_i)$ for $1 \leq i \leq n$;

$$f^*(v_i v_j^{(i)}) = f(v_j^{(i)}) \text{ for } 1 \leq i \leq n, 1 \leq j \leq m_i.$$

Since f is 1-1 and so f^* . Also $\{f^*(E(G))\}$ forms a topology on X^* .

Hence f is a bitopological harmonious labeling and G is a bitopological harmonious graph.

To generate the bitopological harmonious labeling of Shrub Graph $S(m_1, m_2, \dots, m_n)$ using python programming code:

Program:

```

import sys
n=int(input("Enter the value of n\n"))
s = 0
for i in range (1,n+1):
    print("Enter the value of m_",i)
    m_i=int(input())
    if(i==1):
        print("f(v^(1)_1)=empty")
        for j in range (2,m_i+1):
            print("f(v^(1)_"_j,")={1,2,...,"_j-1,"}")
        print("f(v_1)={1,2,...,"_m_i,"}")
        s = s + m_i
    else:
        for j in range(1,m_i+1):
            print("f(v^(_"_i,")_"_j,")={1,2,...,"_s+i-2+j,"}")
        s = s + m_i
    print("f(v^(_"_i,") )={1,2,...,"_s+i-1,"}")
print("f(c)={1,2,...,"_s+n,"}")

```

Output:

```

⇌ Enter the value of n
3
Enter the value of m_ 1
2
f(v^(1)_1)=empty
f(v^(1)_ 2 )={1,2,..., 1 }
f(v_1)={1,2,..., 2 }
Enter the value of m_ 2
3
f(v^( 2 )_ 1 )={1,2,..., 3 }
f(v^( 2 )_ 2 )={1,2,..., 4 }
f(v^( 2 )_ 3 )={1,2,..., 5 }
f(v^( 2 ) )={1,2,..., 6 }
Enter the value of m_ 3
4
f(v^( 3 )_ 1 )={1,2,..., 7 }
f(v^( 3 )_ 2 )={1,2,..., 8 }
f(v^( 3 )_ 3 )={1,2,..., 9 }
f(v^( 3 )_ 4 )={1,2,..., 10 }
f(v^( 3 ) )={1,2,..., 11 }
f(c)={1,2,..., 12 }

```

Example 3.14

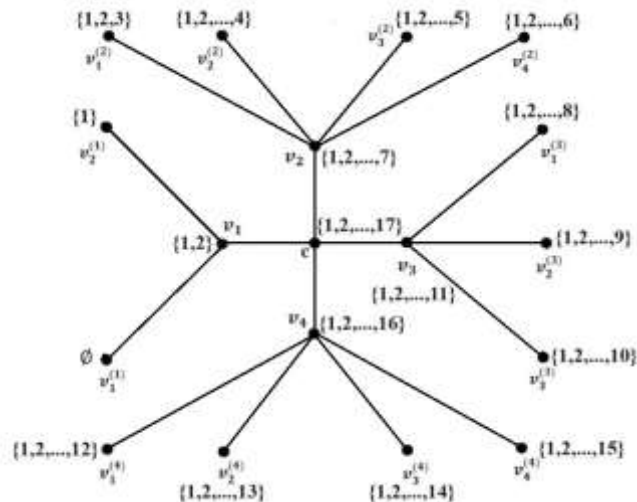


Fig 3.7 Bitopological harmonious labeling of $S(2, 4, 3, 4)$

To generate the bitopological harmonious labeling of above discussed graphs by using python programming code, the link is given as follows:

https://colab.research.google.com/drive/1dXDe_nyKF4MGz37P96nyfvY52ar7oq2g#scrollTo=DkEVcFccO0rM&uniqifier=1

IV. CONCLUSION

In this paper, we proved star graph, path graph, comb graph and shrub graph are bitopological harmonious graph. Further, we labeled the above graphs by using python program code and to give the outputs of the programs.

REFERENCES

- [1] Acharya B.D., Set valuations and their applications, MRI Lecture note in Applied Mathematics, No.2, Mehta Research Institute of Mathematics and Mathematical Physics, 1983.
- [2] Bondy J.A and Murthy U.S.R, "Graph Theory and Application" (North Holland). New York (1976).
- [3] Joseph A Gallian 2018, 'A Dynamic Survey of Graph Labeling', The Electronic Journal of Combinatorics.
- [4] Selestin Lina S, Asha S, 'On Topological Cordial Graphs', Journal of Science and Technology, 5(2020), 25-28.
- [5] Selestin Lina S, Asha S, 'Topological cordial labeling of some graphs', Malaya Journal of Matematik, Vol. 9, No. 1, 861-863.
- [6] Lina, S. & Asha, S. (2022), 'Bitopological labeling of tree related graphs', AIP Conference Proceedings. 2385. 130016. 10.1063/5.0070851.
- [7] G. Siva Prijith, M. Subbulakshmi, S. Chandrakala, 'Topological Cordial Labelling of Some Graphs', Mapana – Journal of Sciences 2023, Vol. 22, Special Issue 1, 129-140 ISSN 0975-3303.