Ulam Stability Of Finite Dimensional Quadratic /Functional Equation In Banach Space And Banach Algebra

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Abstract

In this work, we examine the Ulam stability of finite-dimensional quadratic functional equation (briefly Fun. Equation) in Banach spaces and Banach algebra by utilizing fixed point and direct approaches. Within the context of this quadratic functional equation, as an illustration of the stability of the equation will be regulated by products and sums of powers of norms, we present several instances.

Keywords : Quadratic functional equation, Ulam stability, Fixed point.

1 Introduction and Preliminaries

Since Ulam [24,25] initially posed the issue of the approximate stability of group homomorphisms in 1940, the concept of stability in Fun. Eqs. has grown into a significant field of research. As a result of Hyers [7] in 1941 proof that additive mappings may be stable, the question posed by Ulam was promptly answered, and the idea of Ulam stability was started. Fuzzy and intuitionistic fuzzy spaces are the latest extensions of this generalized theory, which has been applied to metric spaces, Banach spaces, and other functional equations and mathematical frameworks in recent years. Revelations into the estimated characteristics of solutions in complex spaces are gained through studying Ulam stability for various functional equations, which is important for theoretical and applied mathematics.

The outcome of Hyers is made longer by Aoki [1] by assuming that the Cauchy differences are not limited. As shown by Rassias [14, 15], the additive mapping. The result of Rassias was summed up by Gavruta [6].

In [19], the author Skof demonstrated the stability of quadratic fun. eq.

$$g(x_1+x_2)+g(x_1-x_2)=2g(x_1)+2g(x_2) \tag{1.1}$$

in Banach space. Cholewa [3] generalised the Skof's outcome for abelian groups. Then,Skof's [19] result was generalised by many mathematician on many spaces (see [2, 9, 11]).

Numerous researchers have investigated IFN-spaces and IF2N-spaces (Ref. [4, 15, 16, 20]). Numerous scholars have investigated the Ulam-Hyers stability of fun. eqs. in IFN-spaces (see [8, 13, 14, 21–23]).

Since an example of a fun. eq. the quadratic fun. eq. has received a lot of interest. Since quadratic fun. eqs. have applications in dynamical systems and approximation theory, among others, studying their stability is an important mathematical issue. Ulam stability of the quadratic fun. eq. in IFN- spaces and 2-Banach spaces, two frameworks that provide different views on stability, are the main topics of this research. We can study stability in a broader context by using a 2-Banach space, which is an extension of the traditional Banach space. In contrast, IFN-spaces build on the traditional idea of fuzzy spaces with acceptance and rejection levels, offering a more robust mathematical framework to represent the approximation character of findings. Subsequent to this groundwork, substantial strides were achieved in proving stability for quadratic fun. eqs. in broader frameworks such as 2-Banach and IFN-spaces.

In [5], Gahler introduced the notion of 2-Banach spaces has offered a framework for investigating stability in spaces when certain generalised requirements are satisfied by norms. Since then, stability researchers have turned to 2-Banach spaces for their capacity to capture multidimensional norm structures in a flexible and broad way. 2-Banach spaces were recently shown to be useful for exploring the stability of complicated fun. eqs., like the quartic fun. eq. (see. [5,7,10]).

Another way that stability theory has grown is through the use of intuitionistic fuzzy normed spaces. These spaces provide a more complex of stability, especially in domains where approximation and fuzzy logic are important, by adding a new parameter to reflect membership and non-membership degrees. Jung and Rassias showed how IFN-spaces may be used to analyse stability for different types of functional equations. These spaces offer a more robust mathematical framework that can handle the inherent uncertainty and variability in solution sets.

In order to demonstrate that fun. eqs. are stable, the fixed point approach has been utilised extensively. This is due to the fact that Banach's fixed point theorem offers a solid framework for determining the convergence of approximate solutions [7]. Fixed point hypotheses have been shown to be useful in demonstrating Ulam consistency in a variety of mathematical situations, as proved by a number of research, such as those carried out by Radu [18]. Fixed point approaches may not be applicable to fun. eqs., but the direct approach is still an acceptable alternative. By using functional form-specific inequality and approximation, authors can derive stability constraints using the direct method.

In conclusion, although the fixed point and direct approaches have both demonstrated their efficacy in analysing the Ulam stability of fun. eqs., the application of both approaches to 2-Banach and IFN-spaces provides a novel viewpoint on quartic functional equations for the first time. Through the application of both fixed-point and direct approaches, the purpose of this study is to contribute to the existing body of research on functional stability in generalised mathematical spaces. This will be accomplished by conducting a systematic analysis of the Ulam stability of quadratic fun. eqs. in these spaces.

The purpose of this work is to examine the Ulam stability of a finite-dimensional quadratic fun. eq.

$$\sum_{i\leq i<j\leq n} g(s_i+s_j)+\sum_{i\leq i<j\leq n} g(s_i-s_j)=(2n-2)\sum_{i=1}^{n} g(s_i) \tag{1.2}$$

where n ≥ 2, in Banach spaces and Banach algebra by utilizing fixed point and direct approaches. Within the context of this quadratic fun. eq., as an illustration of the stability of the equation can be regulated by products and sums of powers of norms, we present several instances.

Theorem 1.1. [14] Let (G, d) be a generalized complete metric space and a strictly contractive function Ω : G → G with L < 1. Then, for every $v_1 \in G$, either

$$d(\Omega^m v_1,\Omega^{m+1} v_1)=\infty \ ,m\geq m_0 \ ;$$

or there is an interger $m_0 > 0$ fulfills

(i) $d(\Omega^m v_1,\Omega^{m+1} v_1)<\infty \ ,m\geq m_0$;

(ii) the sequence $\{\Omega^{m+1} v_1\}_{m\in N}$ converges to a fixed point v_1^* *of* Ω;

(iii) v_1^* is the only one fixed point of Ω in $G^* =\{v_2 \in G/d(\Omega^{m0} v_1,v_2)<\infty$;

(iv) $d(v_2, v_1^*) \leq \frac{1}{1-L} d(\Omega v_2, v_2), for\, every\;\; v_2 \in G^*$,

Where L is a Lipschitz constant.

2 Solution of (1.2)

Theorem 2.1. If a function g : S → T fulfills the equation (1.2) for every $s_1, s_2\, s_3 .... s_n \in s$, then g : S → T fulfills the equation (1.1) for every s, $t \in S$.

Proof. Let g : S → T fulfills the equation (1.2). Setting ($s_1 = s, s_2 = s, s_3 = ... s_n = 0)$) in (1.2), we get g(0) = 0. Now, setting $(s_1 = s, s_2 = s, s_3 = ... s_n = 0)$ in (1.2), we get g(−s) = g(s) for all $s \in S$. Therefore, g is an even function. Substituting $(s_1 = s, s_2 = s, s_3 = ... s_n = 0)$, we obtain g(2s) =4g(s), and so on for every $s \in S$. . For any positive integer b, we have g(bs) = b^2g(s) for all $s \in S$. Then applying $(s_1 = s, s_2 = s, s_3 = ... s_n = 0)$ in (1.2), we attain (1.1).

Banach Space Stability Results for (1.2)

Let us assume that T is a Banach space and S is a normed space in this context. In order to facilitate notational convenience, we define a function denoted as Dg : S → T.

$$Dg(s_1, s_2, ... s_n) = \sum_{i \leq i < j \leq n} g(s_i + s_j) + \sum_{i \leq i < j \leq n} g(s_i - s_j) - (2n-2) \sum_{i=1}^{n} g(s_i)$$

for every $s_1, s_2\, s_3 .... s_n \in s$

2.1 Stability of (1.2): Direct Method

Theorem 2.2.

Let $j = \pm 1$. Let a function $\alpha : S^n \to [0, \infty)$ Satisfies

$$\lim_{k \to \infty} \frac{\alpha(2^{kj} s_1, 2^{kj} s_3, ..., 2^{kj} s_n)}{2^{2kj}} = 0 \tag{2.1}$$

For every $s_1, s_2\, s_3 .... s_n \in s$. Let $Dg : S \to T$ be a function fulfilling

$$\| Dg(s_1, s_2, ... s_n) \| \leq \alpha(s_1, s_2, ... s_n) \tag{2.2}$$

For every $s_1, s_2\, s_3 .... s_n \in S$ then there exits only one quadratic function: $Q : S \to T$ satisfying

$$\| g(s) - Q(s) \| \leq \frac{1}{2(n-1)} \sum_{k = \frac{1-j}{2}} \frac{\mu(2^k s)}{2^{2k}} \tag{2.3}$$

where $\mu(s) = \frac{\alpha(s, s, s, ... s\;)}{n}$ (2.4)

for every $s \in S$. The mapping Q(s) is defined by

$$Q(s) = \lim_{l \to \infty} \frac{g(2^l s)}{2^{2l}} \tag{2.5}$$

for every $s \in S$.

Proof

Assume that j=+1. Setting $(s_1 = s_2 = s_3 = \ldots\ldots s)$ in (2.2) , we arrive

$$\left\| 2n(n-1)\,g(s) - \frac{(n^2-2)g(2s)}{2} \right\| \leq \alpha\,(s,s,\ldots s\ ) \tag{2.6}$$

for every $s \in S$. From (2.6) that

$$\left\| \frac{g(2s)}{2^2} - g(s) \right\| \leq \frac{\alpha\,(s,s,\ldots s\ )}{2n(n-1)} \tag{2.7}$$

for every $s \in S$. Switching s by 2s and dividing by 2^2 in (2.7) , we reach

$$\left\| \frac{g(2^2 s)}{2^4} - \frac{g(2s)}{2^2} \right\| \leq \frac{\mu(2s)}{2^2.2(n-1)} \tag{2.8}$$

for every $s \in S$. Adding (2.7) and (2.8) , we obtain

$$\left\| \frac{g(2^2 s)}{2^4} - g(s) \right\| \leq \frac{1}{2(n-1)}\left[\mu(s) + \frac{\mu(2s)}{2^2} \right]$$

for every $s \in S$. For any integer $l > 0$, We generalized that

$$\left\| \frac{g(2^l s)}{2^{2l}} - g(s) \right\| \leq \frac{1}{2(n-1)} \sum_{k=0}^{l-1} \frac{\mu(2^k s)}{2^{2k}}$$

$$\left\| \frac{g(2^l s)}{2^{2l}} - g(s) \right\| \leq \frac{1}{2(n-1)} \sum_{k=0}^{\infty} \frac{\mu(2^k s)}{2^{2k}} \tag{2.9}$$

for every $s \in S$. To show that $\frac{g(2^l s)}{2^{2l}}$ is convergence sequence , switching s by $2^m s$ and dividing 2^{2m} in (2.9) , for $q,m > 0$, we arrive $\left\| \frac{g(2^{q+m} s)}{2^{2(q+m)}} - \frac{g(2^m s)}{2^{2m}} \right\| \leq \frac{1}{(2n-2)} \sum_{k=0}^{i-1} \frac{\mu(2^{k+m} s)}{2^{2(k+m)}} \to 0 \, as\ m \to \infty$ (2.10)

for every $s \in S$. Thus $\frac{g(2^q s)}{2^{2q}}$ is a Cauchy sequence. As the result, T is complete , there is a function $Q : S \to T$ fulfilling $Q(s) = \lim_{l\to\infty} \frac{g(2^q s)}{2^{2q}}$

for every $s \in S$. Taking limit $q \to \infty$ in(2.9) , we obtain (2.4) valid for every $s \in S$

To show that Q fulfills(1.2) , switching $s_1, s_2, s_3 \ldots\ldots s_n$ by $2^{2m}s, 2^{2m}s, 2^{2m}s \ldots\ldots 2^{2m}s$ and dividing $2^{2^{2m}}$ in (2.2), we reach

$$\frac{1}{2^{2m}} \left\| Dg(2^{2m}s, 2^{2m}s, 2^{2m}s, \ldots\ldots, 2^{2m}s) \right\| \leq \frac{1}{2^{2m}}\, \alpha\,(2^{2m}s, 2^{2m}s, 2^{2m}s, \ldots\ldots, 2^{2m}s)$$

for every $s_1, s_2, \ldots, s_n \in S$. Taking the limit $m \to \infty$ in above inequality, we obtain that $Q(s_1, s_2, \ldots, s_n) = 0$. Hence Q satisfies (1.2) for every $s_1, s_2, \ldots, s_n \in S$. To show that Q is the only one solution. Let R(s) be the another quadratic solution which fulfilling (1.2) and (2.4), then

$$\| Q(s) - R(s) \| \leq \frac{1}{2^{2m}} \{\| Q(2^m s) - g(2^m s) \| + \| g(2^m s) - R(2^m s) \|\}$$

$$\leq \frac{1}{2(n-1)} \sum_{k=0}^{\infty} \frac{\mu(2^{k+m} s)}{2^{2(k+m)}} \to 0 \text{ as } m \to \infty$$

for every $s \in S$.Hence Q is the only one solution. For $j = -1$, we can demonstrate an equivalent stability finding.

Corollary 2.3. [10] If a mapping $Dg: S \to T$ satisfies

$$\|Dg(s_1, s_2, \dots, s_n)\| \leq \begin{cases} \rho, \\ \rho\{\sum_{\tau=1}^{n} \|s_\tau\|^a\}, \\ \rho\{\prod_{\tau=1}^{n} \|s_\tau\|^a\}, \\ \rho\{\prod_{\tau=1}^{n} \|s_\tau\|^a + \sum_{\tau=1}^{n} \|s_\tau\|^{na}\}. \end{cases} \tag{2.11}$$

for every $s_1, s_2, \dots, s_n \in S$. Then there is only one quadratic function $Q: S \to T$ fulfilling

$$\|g(s) - Q(s)\| \leq \begin{cases} \frac{2\rho}{3n|(n-1)|} \\ \frac{2\rho\|s\|^a}{n(n-1)|2^2-2^a|}; & a \neq 2 \\ \frac{2\rho\|s\|^{an}}{n(n-1)|2^2-2^{an}|}; & a \neq \frac{2}{n} \\ \frac{2(n+1)\rho\|s\|^{an}}{n(n-1)|2^2-2^a n|}; & a \neq \frac{2}{n}. \end{cases} \tag{2.12}$$

for every $s \in S$, where ρ and a are in $\mathbb{R}^+$.

2.2 Stability of (1.2): Fixed Point Technique

Theorem 2.4. Let a mapping $Dg: S \to T$ for which there is a function$\alpha: S^n \to [0, \infty)$ with

$$\lim_{k \to \infty} \frac{\alpha(\eta_i^k s_1, \eta_i^k s_2, \dots, \eta_i^k s_n)}{\eta_i^{2k}} = 0 \tag{2.13}$$

where$\eta_i = \begin{cases} 2 \text{ if } i = 0; \\ \frac{1}{2} \text{ if } i = 1; \end{cases}$ fulfilling

$$\|Dg(s_1, s_2, s_3, \dots, s_n)\| \leq \alpha(s_1, s_2, s_3, \dots, s_n) \tag{2.14}$$

for every $s_1, s_2, s_3, \dots, s_n \in S$. If there is $L = L(i)$ fulfills

$$\gamma(s) = \frac{1}{n(n-1)} \alpha\left(\frac{s}{2}, \frac{s}{2}, \dots, \frac{s}{2}\right)$$

has the property

$$\frac{\gamma(\eta_i s)}{\eta_i^2} = L\gamma(s) \tag{2.15}$$

for every $s \in S$, then there exists only one quadratic function $Q: S \to T$ satisfying (1.2) and

$$\|g(s) - Q(s)\| \leq \frac{L^{1-i}}{1-L} \gamma(s) \tag{2.16}$$

for every s ∈ S .

Corollary 2.5.[10]

If a function $Dg: S \to T$ *Such that*

$$\| Dg(s_1, s_2, ...s_n) \leq \left\{ \begin{array}{c} \rho \\ \rho\left\{ \sum_{T=1}^{n} \|s_T\|^a \right\}, \\ \rho\left\{ \prod_{T=1}^{n} \|s_T\|^a \right\}, \\ \rho\left\{ \prod_{T=1}^{n} \|s_T\|^a \right\} + \sum_{r=1}^{n} \|s_T\|^{na} \quad \} \end{array} \right. \tag{2.17}$$

For every $s_1, s_2\ s_3....s_n \in s$ .Then there is only one quadratic solutiont $Q: S \to T$ fulfilling

$$\| g(s) - Q(s) \ \| \leq \left\{ \begin{array}{l} \dfrac{2\rho}{3n|n-1|} \\ \dfrac{2\rho\|s\|^a}{n(n-1)|2^2 - 2^a|};\ a \neq 2 \\ \dfrac{2\rho\|s\|^{an}}{n(n-1)|2^2 - 2^{an}|};\ a \neq \dfrac{2}{n} \\ \dfrac{2(n+1)\rho\|s\|^{an}}{n(n-1)|2^2 - 2^{an}|};\ a \neq \dfrac{2}{n} \end{array} \right. \tag{2.18}$$

For every $s \in S$ .where ρ *and* a *are in* R^+.

Banach Algebra Stability Results for (1.2)

Here , we Consider S is a normed algebra and T is Banach algebra.

2.3.Stability of(1.2) –Direct Technique

Definition 2.6

Let S be Banach Algebra. A function $g: S \to S$ is known as quadratic derivation

If the function g fulfills,

$g(s_1 s_2) = g(s_1) s_2^2) + s_1^2 g(s_2)$ for every $s_1, s_2 \in S$ .Also the quadratic derivation for n-variables fulfills $g(s_1, s_2, ...s_n) = g(s_1) s_2^2 ......s_n^2 + s_1^2 g(s_2) s_3^2 ......s_n^2 + ... + s_1^2 s_2^2 ......g(s_n)$

For every $s_1, s_2\ s_3....s_n \in s$.

Proposition 2.7.[10] Let $j = \pm 1$ .Let a mapping $Dg: S \to T$ for which there is function $\alpha, \beta: S^n \to [0, \infty)$ with

$\sum_{k=o}^{\infty} \frac{\alpha(2^{kj} s_1, 2^{kj} s_2, ..., 2^{kj} s_n)}{2^{2kj}}$ converges in R and $\lim_{k\to\infty} \frac{\alpha(2^{kj} s_1, 2^{kj} s_2, ..., 2^{kj} s_n)}{2^{2kj}} = 0$ and also

$\sum_{k=o}^{\infty} \frac{\beta(2^{kj} s_1, 2^{kj} s_2, ..., 2^{kj} s_n)}{2^{2kj}}$ converges in R and $\lim_{k\to\infty} \frac{\beta(2^{kj} s_1, 2^{kj} s_2, ..., 2^{kj} s_n)}{2^{2kj}} = 0$ and also

$$\|Dg(s_1,s_2,s_3,...s_n)\| \le \alpha((s_1,s_2,s_3,...s_n) \text{ and}$$

$$\left\|Dg(s_1,s_2,s_3,...s_n) - Dg(s_1)\ s_2^2.....s_n^2 - s_1^2 Dg(s_2)\ s_3^2....s_n^2\ - .....- s_1^2 s_2^2\ s_3^2....s_n^2 Dg(s_n)\right\| \le \beta((s_1,s_2,s_3,...s_n)$$

for every $s_1,s_2,...s_n \in S$, then there exits only one quadratic derivation mapping $Q : S \to T$

Satisfying (1.2) and $\| g(s) - Q(s)\ \| \le \dfrac{1}{2(n-1)} \displaystyle\sum_{k=\frac{1-j}{2}} \dfrac{\mu(2^k s)}{2^{2k}}$

where $\mu(s) = \dfrac{\alpha(s,s,s,...s\ )}{n}$

for every $s \in S$. The mapping Q(s) is defined by

$$Q(s)\ = \lim_{k\to\infty} \frac{g(2^{kj}s)}{2^{2kj}}$$

for every $s \in S$.

Corollary 2.8.

Let the function $Dg : S \to T$ *such that*

$$\| Dg(s_1,s_2,.....s_n) \le \left\{ \begin{array}{c} \rho \\ \rho\left\{\displaystyle\sum_{T=1}^{n} \|s_T\|^a\right\}, \\ \rho\left\{\displaystyle\prod_{T=1}^{n} \|s_T\|^a\right\}, \\ \rho\left\{\displaystyle\prod_{T=1}^{n} \|s_T\|^a\right\} + \displaystyle\sum_{r=1}^{n} \|s_T\|^{na}\quad \} \end{array} \right.$$

and

$$\left\|Dg(s_1,s_2,s_3,...s_n) - Dg(s_1)\ s_2^2.....s_n^2 - s_1^2 Dg(s_2)\ s_3^2....s_n^2\ - .....- s_1^2 s_2^2\ s_3^2....s_n^2 Dg(s_n)\right\| \le \left\{ \begin{array}{c} \rho \\ \rho\left\{\displaystyle\sum_{T=1}^{n} \|s_T\|^a\right\}, \\ \rho\left\{\displaystyle\prod_{T=1}^{n} \|s_T\|^a\right\}, \\ \rho\left\{\displaystyle\prod_{T=1}^{n} \|s_T\|^a\right\} + \displaystyle\sum_{r=1}^{n} \|s_T\|^{na}\quad \} \end{array} \right.$$

for every $s_1,s_2,...s_n \in S$, then there exits only one quadratic derivation mapping $Q : S \to T$

fulfilling $\| g(s)-Q(s) \| \le \begin{cases} \dfrac{2\rho}{3n|n-1|} \\ \dfrac{2\rho\|s\|^{a}}{n(n-1)\left|2^{2}-2^{a}\right|};\ a\neq 2 \\ \dfrac{2\rho\|s\|^{an}}{n(n-1)\left|2^{2}-2^{an}\right|};\ a\neq \dfrac{2}{n} \\ \dfrac{2(n+1)\rho\|s\|^{an}}{n(n-1)\left|2^{2}-2^{an}\right|};\ a\neq \dfrac{2}{n} \end{cases}$

for every $s \in S$ where ρ *and* α *are in* $R+$.

2.4.Stability of (1.2) : Fixed point Technique

Proposition 2.9. Let $j=\pm 1$ .Let a mapping $Dg: S \rightarrow T$ for which there is function $\alpha, \beta : S^{n} \rightarrow [0,\infty)$ with

$\sum_{k=o}^{\infty} \dfrac{\alpha(2^{kj}s_1, 2^{kj}s_2,......,2^{kj}s_n)}{2^{2kj}}$ converges in R and $\lim_{k\to\infty} \dfrac{\alpha(2^{kj}s_1, 2^{kj}s_2,......,2^{kj}s_n)}{2^{2kj}}=0$ and also

$\sum_{k=o}^{\infty} \dfrac{\beta(2^{kj}s_1, 2^{kj}s_2,......,2^{kj}s_n)}{2^{2kj}}$ converges in R and $\lim_{k\to\infty} \dfrac{\beta(2^{kj}s_1, 2^{kj}s_2,......,2^{kj}s_n)}{2^{2kj}}=0$ and also

$\|Dg(s_1,s_2,s_3,...s_n)\| \le \alpha((s_1,s_2,s_3,...s_n)$ and

$\|Dg(s_1,s_2,s_3,...s_n) - Dg(s_1)\ s_2^2.....s_n^2 - s_1^2 Dg(s_2)\ s_3^2....s_n^2\ - .....- s_1^2 s_2^2\ s_3^2....s_n^2 Dg(s_n)\| \le \beta((s_1,s_2,s_3,...s_n)$

for every $s_1,s_2,...s_n \in S$.

Then there is $L = L(i) < 1$ fulfilling

$\gamma(s) = \dfrac{1}{n(n-1)}\alpha\left(\dfrac{s}{2},\dfrac{s}{2},....\dfrac{s}{2}\right)$ has the property $\dfrac{\beta(\eta_i s)}{\eta_i^{2}} = L\,\beta(s)$ for every $s \in S$, then there exits only one quadratic solution $Q: S \rightarrow T$ satisfying (1.2) and $\| g(s)-Q(s) \| \le \dfrac{L^{1-i}}{1-L}\beta(s)$ for every $s \in S$.

Corollary 2.10

Let the function $Dg: X \rightarrow Y$ *such that*

$\| Dg(s_1,s_2,.....s_n) \le \begin{cases} \rho \\ \rho\left\{\sum_{T=1}^{n}\|s_i\|^{a}\right\}, \\ \rho\left\{\prod_{T=1}^{n}\|s_i\|^{a}\right\}, \\ \rho\left\{\prod_{T=1}^{n}\|s_i\|^{a}\right\}+\sum_{r=1}^{n}\|s_i\|^{na}\ \ \} \end{cases}$

and

$$\left\| Dg(s_1, s_2, s_3, \dots s_n) - Dg(s_1)\ s_2^2 \dots\dots s_n^2 - s_1^2 Dg(s_2)\ s_3^2 \dots\dots s_n^2\ - \dots\dots - s_1^2 s_2^2\ s_3^2 \dots\dots s_n^2 Dg(s_n) \right\| \le \begin{cases} \rho \\ \rho\left\{ \sum_{T=1}^{n} \|s_i\|^a \right\}, \\ \rho\left\{ \prod_{T=1}^{n} \|s_i\|^a \right\}, \\ \rho\left\{ \prod_{T=1}^{n} \|s_i\|^a \right\} + \sum_{r=1}^{n} \|s_i\|^{na} \quad \} \end{cases}$$

for every $s_1, s_2, \dots s_n \in S$, then there exits only one quadratic derivation mapping $Q : S \to T$

fulfilling $\| g(s) - Q(s) \| \le \begin{cases} \dfrac{2\rho}{3n|n-1|} \\ \dfrac{2\rho\|s\|^a}{n(n-1)|2^2 - 2^a|};\ a \neq 2 \\ \dfrac{2\rho\|s\|^{an}}{n(n-1)|2^2 - 2^{an}|};\ a \neq \dfrac{2}{n} \\ \dfrac{2(n+1)\rho\|s\|^{an}}{n(n-1)|2^2 - 2^{an}|};\ a \neq \dfrac{2}{n} \end{cases}$

for every $s \in S$ where $\rho\ and\ \alpha\ are\ in\ R+$.

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