Construction And Analysis Of Hyper Block Graeco Latin Sudoku Square Design

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ABSTRACT

A novel experimental design called Hyper Block Graeco Latin Sudoku Square Design (Hyper Block GLaSS Design) has been released. It applies the row blocking and column blocking features of Subramani and Pormuswamy Sudoku Square Designs. The Error Sum of Square is decreased by implementing the Block Sum of Square for both rows and columns. Hyper Block GLaSS Design aims to investigate eight parameters and test three sets of treatments concurrently in a single experiment. For Hyper Block GLaSS Design a numerical example is used to analyze the fixed effect model's construction, compare it to the Hyper Graeco Latin Sudoku Square Design, and determine how efficient it is.
By adding row and column blocking, the suggested new design outperforms the Hyper Graeco Latin Sudoku Square Design in terms of lowest mean squares error.

KEYWORDS Hyper Graeco Latin Sudoku Square Design (HGD) Hyper Block GLaSS Design Sudoku Design(HBGD).

1. INTRODUCTION

Sudoku is a logical puzzle game with no mathematics involved in it. This game usually comprises of nine-by-nine grid in which there are some blank cells while other are filled by numbers, colors or letters. The blank cells are filled in a manner that every single row, every single column and each grid (3x3 blocks) comprises of all the nine digits just once (Bailey et al. 2008).
Subramani and Pormuswamy (2009) pioneered the introduction of a innovative experimental design, known as Sudoku Design, and conducted an exhaustive examination of its construction, analysis, and practical application.

The primary goal of researchers in the experimental sector is; and they are very interested in minimizing experimental errors and in obtaining as much information as possible with the least amount of resources (Kirk, 1982). Although a good design has the least amount of experimental error, a perfect design is still unattainable because of various circumstances and environmental factors (Lakic, 2002).
For the sole purpose of lowering error, researchers continue to attempt to offer novel designs (Sorana et al, 2009).Comparative designs such as Latin Square, Graeco Latin Square, and Hyper Graeco Latin Squares are used in experimental fields where variance for columns and rows needs to be controlled (Cocran, 1974).
It is the well-organized design in the existence of irritant factors from four sides with one treatment (Colbourn and Dinitz, 2006). With intriguing rules, Sudoku is an engrossing number assignment problem. The Sudoku problem contains of a 9-by-9 grid with the numbers 1 through 9 filled in so that each row, column and all of the grid 3x3 nine blocks can only contain one of the nine digits. Although it was not invented in Japan, Sudoku is a combination of two Japanese words, with the word Su standing for digit and Doku meaning only one (Okagbue et al., 2015).The Sudoku design, a unique kind of Latin square created by Garns in 1979 and based on Sudoku puzzles, is widely utilized by people worldwide in design and electrical media (Bailey et al., 2008). The entire experimental part of (n_1 x n_2) plots is separated into (n_1 x n_2) regions, each of which has (n_1 x n_2) plots and treatments applied to these plots.

These treatments are ordered so that there are no frequent numbers in column or row. While Sudoku grids are fully designed in Sudoku design, the blank blocks in Sudoku grids are referred to as Sudoku puzzles Ponnuswamy and Subramani (2009). A very basic approach was suggested by Fontana (2014) aimed at the identical unsystematic sampling of Sudoku and Latin square designs. The algorithm could be executed using the complete network up to order n, but it needed the greatest cliques of the proper graph. Aslam (2008) pointed out that the popular number placement puzzle known as the Sudoku puzzle was linked to the word Sudoku. This contains (n_1x n_2) experimental units and (n_1x n_2) treatments applied to these experimental parts so that each treatment appears once in every block or square, once in every row, and once in every column. The $(n_1 \text{ x } n_2)^2$ grid is divided into (n_1 x n_2) spatially compact squares or blocks. This puzzle comprises of a 9x9 web which is distributed into 3×3 regions or blocks, must contain only one number from 1 to 9.

Overlapping three varieties of OLSD (Orthogonal Latin Square Design) with three distinct treatment types results in HGLSD (Hyper-Graeco Latin Square Design. It may test three sets of treatments at the same time in the same experiment, allowing for the simultaneous study of five factors: three treatments, rows, and columns (Colbourn and Dinitz, 2006). The treatments applied to these plots are ordered so that there are no repeated numbers in any row or column. Whereas Sudoku grids are complete in Sudoku design, the blank blocks in Sudoku grids are referred to as Sudoku puzzles (Subramani and Ponnuswamy, 2009). Blocks can be added to designs to improve their accuracy (Sorana et al., 2009). An effort has been made to present a new efficient design, Hyper Graeco Latin Sudoku Square Design (Hyper GLaSS Design), by adding a blocking property of Sudoku Square Design to Hyper Graeco Latin Square Design in an effort to further reduce the mean square error.
Chungen (2009) used the right concepts and algorithms to create Sudoku puzzles with different levels of complexity. The framework, analysis, and presentations of Sudoku square design were given by Subramani and Pormuswamy (2009) using statistical examples. XuX et al., (2011) created a design that may be further subdivided into groups to create sub designs. Therefore, these comprehensive designs might provide the highest level of uniformity in univariate and bi-variate margins. A straightforward technique for uniform random selection of Sudoku and Latin square designs was created by Fontana (2013).

Hussain et al., (2017) introduced a novel experimental design known as the Hyper Graeco Latin Sudoku Square Design, or Hyper GLaSS Design, in which the blocking property of the Sudoku square design was employed to provide more effective outcomes and Hyper GLaSS design enabled the simultaneous examination of three sets of treatments, allowing for the investigation of six factors in a single study. The Parameter estimation under mixed and random effects models was made easier by this creative design. Hussain et al., (2019) demonstrated the analysis of the Hyper GLaSS design using a numerical example with a hypothetical dataset, showcasing its potential for efficient and comprehensive experimentation. The simulation results had revealed that the proposed design outperformed the Hyper Graeco Latin Square design in terms of efficiency, thus highlighting its potential for improved experimental outcomes.

Objective of the research study are:

1. To create Hyper Block Graeco Latin Sudoku Square design layout by introducing the row and column blocking property of Subramani in Hyper GLaSS design and associate Hyper Block Greaco Latin Sudoku Square design with Hyper GLaSS design
2. Develop Hyper Block GLaSS design the fixed effect model, analysis, and relative efficiency are examined and contrasted with Hyper Graeco Latin Sudoku Square Design using numerical examples.

1. HYPER Block GLaSS DESIGN CONSTRUCTION

The experimental units are arranged in "m^2" rows, "m^2" columns, "m^2" blocks, and "m^2" squares in the Hyper Block GLaSS design (HBGD), a unique variation of the Hyper Graeco Latin soduko square design with additional restrictions.
These experimental units are given several treatment kinds so that each treatment must appear only once in each row, once in each column, and once in each region or block. For the Hyper Block GLaSS design

The Hyper Block Glass Design is the new experimental design develop from the Hyper GLaSS design as row blocking and column blocking is introduced in the hyper GLaSS design which reduced the error. We can test three sets of treatments at the same time in a single experimentation. Eight factors (rows, columns, square row blocks, column blocks, n_1 n_2 runs, analysis of treatments types (1), (2), and (3) are possible at n levels.

(HBGD) architecture of any order (even or odd), the experimental area must be divided both vertically and horizontally.

The horizontal division is called Horizontal Grids, while the vertical divide is called Vertical Grids. Starting from column 1 to column (m-1)m+1, the Vertical Grids for treatment type (1) comprise of 1 to m4 digits in a matrix form in a sequential manner. The second column begins from m-1 to (m-1)m+2, and so on. In a similar vein, the Horizontal Grids have 1 to m2 digits arranged so that the first row begins at 1 and ends at m, the second row begins at m+1 and ends at 2 m, and so on.

The suggested design i-e Hyper Block GLaSS design (Hyper Block Graeco Latin Sudoku Square design)provide more opportunities than hyper GLaSS design in the reduction of error through proper procedure

2. LIMITATIONS OF HYPER Block GLaSS DESIGN

The HBGD Design's blocks as a blocking factor are only successful if the variance between row and column blocks is substantially greater than the variance within row and column blocks; otherwise, the suitable Because the loss of degrees of freedom will raise the error mean square and render the results inconsequential, the Hyper-Graeco-Latin Square Design might be the best option. The blocks for the rows and columns are square in shape. The study does not include an interaction situation. This study's suggested ANOVA is predicated on the idea that the errors are uncorrelated. When $m \geq 6$, the Hyper Block GLaSS Design is beneficial.

Table 1 shows the HBG Design layout for Straight and Strange Order (see appendix).

Example:

Let the first row of the type (1) treatments Sudoku square design is { A, E, I, M, B, F, J, N, C, G, K, O, D, H ,L and P}Then the first row of its orthogonal treatments (2) type Sudoku square design is { A, B, C, D, E, F, G, H, I ,J, K , L, M, N ,O, and P } and the first row of its third orthogonal treatments type (3) Sudoku square design is {P, L, H, D, O, K, G, C, N, J, F, B, M, I, E and A}. Table 2 shows the comprehensive Sudoku square design of treatments of (1) type with the original row {A, E, I, M, B, F, J, N, C, G, K, O, D, H, L, and P} as; (see appendix

Note: The suggested consecutive method of constructing Hyper Block GLaSS design is effective for any value of m^2 (odd or even).

3. HYPER-BLOCK GLaSS DESIGN STATISTICAL ANALYSIS

The treatments of three type i.e. treatment 1, treatment 2 and treatment 3 tested in $m^2.m^2$ Sudoku Square design with m^2 columns, m^2 rows and m^2 blocks. Statistical analysis of permanent effect model without any interaction term is as;

$y_{kl(rpqstu)}$ is the experimental field having k^{th} row, l^{th} column, r^{th} treatments of 1^{st} type, p^{th} block, q^{th} treatments of 2^{nd} type and s^{th} treatment of 3^{rd} type t^{th} row block and u^{th} column block are applied.

The model of HBG Design is

$$Y_{kl(rpqstu)} = \mu + \alpha_k + \beta_l + \tau_r + \gamma_p + \theta_q + \lambda_s + \varphi_t + \omega_u + \varepsilon_{kl(rpqstu)}$$

$$k,l,r,p,q,s,t,u = 1,2,3..........m^2$$

Where,

$\mu =$ population mean

$\alpha_k =$ k^{th} row effect ,

$\beta_l =$ l^{th} column effect

τ_r = rth treatments effect of 1 type

γ_p = pth block effect,

θ_q =effect of the qth treatments of type (2)

λ_s =effect of the sth treatments type (3)

φ_t = effect of the tth row block

ω_u = effect of the uth column block

$\varepsilon_{kl(rpqstu)}$ = effect of random error

The limitations are $\Sigma\alpha_k = \Sigma\beta_l = \Sigma\tau_r = \Sigma\gamma_p = \Sigma\theta_q = \Sigma\lambda_s = \Sigma\varphi_t = \Sigma\omega_u = 0$

With the molds that $y_{kl(rpqstu)}$ is direct function, and $\mu, \alpha_k, \beta_l, \tau_r, \gamma_p, \theta_q, \lambda_s, \varphi_t, \omega_u, \varepsilon_{kl(rpqstu)}$ are identically and individually distributed as $N\left(0, \sigma^2\right)$.

The symbolizations used in the statistical analysis of HBGD are following:

$$\sum_{l=1}^{m} y_{kl(rpqstu)} = R_k = k^{th} \text{ row total}$$

$$\sum_{k=1}^{m^2} y_{kl(rpqstu)} = C_l = l^{th} \text{ column total}$$

$$\sum_{p=1}^{m^2} y_{kl(rpqstu)} = T_r = r^{th} \text{ treatments of type (1) total}$$

$$\sum_{p=1}^{m^2} y_{kl(rpqstu)} = B_p = p^{th} \text{ block total}$$

$$\sum_{q=1}^{m^2} y_{kl(rpqstu)} = \theta_q = q^{th} \text{ treatment of type (2) total}$$

$$\sum_{t=1}^{m^2} y_{kl(rpqstu)} = \lambda_s = s^{th} \text{ treatment of type (3) total}$$

$$\sum_{u=1}^{m} y_{kl(rpqstu)} = \varphi_t = t^{th} \text{ row block total}$$

$$\sum_{r=1}^{m} y_{kl(rpqstu)} = \omega_u = u^{th} \text{ column block total}$$

Then,

$$\bar{Y}_{k.(......)} = \bar{Y}_k = \frac{R_k}{m^2} = \text{i}^{\text{th}} \text{ row mean}$$

$$\bar{Y}_{.l(......)} = \bar{Y}_l = \frac{C_l}{m^2} = \text{l}^{\text{th}} \text{ column mean}$$

$$\bar{Y}_{..(r.....)} = \bar{Y}_r = \frac{T_r}{m^2} = \text{r}^{\text{th}} \text{ treatments of type (1) mean}$$

$$\bar{Y}_{..(.p....)} = \bar{Y}_p = \frac{B_p}{m^2} = \text{p}^{\text{th}} \text{ block mean}$$

$$\bar{Y}_{..(..q...)} = \bar{Y}_q = \frac{\theta_q}{m^2} = \text{q}^{\text{th}} \text{ treatments of type (2) mean}$$

$$\bar{Y}_{..(...s..)} = \bar{Y}_s = \frac{\lambda_s}{m^2} = \text{s}^{\text{th}} \text{ treatments type (3) mean}$$

$$\bar{Y}_{..(....t.)} = \bar{Y}_t = \frac{\varphi_t}{m^3} = \text{t}^{\text{th}} \text{ row block total}$$

$$\bar{Y}_{..(......u)} = \bar{Y}_u = \frac{\omega_u}{m^3} = \text{u}^{\text{th}} \text{ column block total}$$

Where, kth row, lth column, rth treatment of type 1, pth block, qth treatment of type 2, sth treatment of type 3 are from 1 to m^2 tth row block and tth column block are from 1 to m.

$$\bar{Y} = \frac{G}{m^4} = \text{Grand mean}$$

$$C.F. = \frac{G^2}{m^4} = \text{Correction factor}$$

$$\text{Total Sum of Square} = \text{TSS} = \sum_{k=1}^{m^2}\sum_{l=1}^{m^2}\left(y_{kl(rpqstu)} - \bar{Y}\right)^2 = \sum_{k=1}^{m^2}\sum_{l=1}^{m^2} y^2_{kl(rpqstu)} - C.F.$$

with $m^2 - 1$ d.f

$$\text{Rows Sum of Square} = \text{SSR} = m^2\sum_{k=1}^{m^2}\left(\bar{Y}_k - \bar{Y}\right)^2 = \frac{1}{m^2}\sum_{k=1}^{m^2} R_k^2 - C.F.$$,

with m^2-1 d.f

$$\text{Columns Sum of Square} = \text{SSC} = m^2 \sum_{l=1}^{m^2} \left(\overline{Y_l} - \overline{Y} \right)^2 = \frac{1}{m^2} \sum_{l=1}^{m^2} C_l^{\,2} - C.F. \quad ,$$

with m^2-1 d.f

$$\text{Treatment (1) Sum of Square=SST(1)} = m^2 \sum_{r=1}^{m^2} (\overline{Y}_r - \overline{Y})^2 = \frac{1}{m^2} \sum_{r=1}^{m^2} T_r^{\,2} - C.F.$$

with m^2-1 d.f

$$\text{Blocks Sum of Square} = \text{SSB} = m^2 \sum_{p=1}^{m^2} \left(\overline{Y_p} - \overline{Y} \right)^2 = \frac{1}{m^2} \sum_{l=1}^{m^2} B_l^{\,2} - C.F. \quad ,$$

with m^2-1 d.f

$$\text{Treatment (2) Sum of Square=SST(2)} = m^2 \sum_{q=1}^{m^2} \left(\overline{Y_q} - \overline{Y} \right)^2 = \frac{1}{m^2} \sum_{q=1}^{m^2} \theta_q^{\,2} - C.F.$$

with m^2-1 d.f

$$3^{rd}\ \text{Treatment Sum of Square=SST(3)} = m^2 \sum_{s=1}^{m^2} \left(\overline{Y_s} - \overline{Y} \right)^2 = \frac{1}{m^2} \sum_{s=1}^{m^2} \lambda_s^{\,2} - C.F. \qquad \text{with } m^2\text{-1 d.f}$$

$$\text{Row Block Sum of Square=SSRB} = m \sum_{t=1}^{m} \left(\overline{Y_t} - \overline{Y} \right)^2 = \frac{1}{m^3} \sum_{t=1}^{m} \varphi_t^{\,2} - C.F. \qquad \text{with m-1 d.f}$$

$$\text{Column Block Sum of Square=SSCB} = m \sum_{u=1}^{m} \left(\overline{Y_u} - \overline{Y} \right)^2 = \frac{1}{m^3} \sum_{u=1}^{m} \omega_u^{\,2} - C.F. \qquad \text{with m-1 d.}$$

$$\text{SSE} = \text{Error Sum of Square} = \sum_{k=1}^{m^2} \sum_{l=1}^{m^2} \left(\sum_{r=1}^{m^2} \sum_{p=1}^{m^2} \sum_{q=1}^{m^2} \sum_{s=1}^{m^2} \sum_{t=1}^{m} \sum_{u=1}^{m} \right) \left(Y_{\text{kl}(rpqstu)} - \overline{Y}_k - \overline{Y}_l - \overline{Y}_r - \overline{Y}_p - \overline{Y}_q - \overline{Y}_s - \overline{Y}_t - \overline{Y}_u + 7Y \right)^2$$

$$with\ (m-1)\left[(m+1)(m^2-7) - 2 \right] d.f$$

These results are potted in Table 6 as; (see appendix)

5. NUMERICAL ILLUSTRATION FOR COMPARISON

Hyper Block GLaSS Designs analysis and its comparison with Hyper Graeco Latin Sudoku Square Design through hypothetical data is in Table 7 as; (see appendix)
The ANOVA of Hyper block Graeco Latin Sudoku Square Design is in Table 8 as; (see appendix)
The matching ANOVA Table for the Hyper-Block GLaSS Design is in Table 9 as; (see appendix)

6. HYPER BLOCK GRAECO LATIN SODUKO SQUARE Design vs. HYPER GLASSS DESIGN: A COMPARISON OF EFFICACY

The estimated relative efficiency (RE) derived from the relation is a more accurate and quantifiable indicator of the effectiveness of the Hyper Block GLaSS design over the Hyper GLaSS design.

Comparative efficiency $= E_2 \,/\, E_1$

The relative efficiency of HGD in comparison to HBGD may be found by calculating the ratio between E1 and E2, as E1 represents the error mean square of HGD and E2 represents the error mean square of HBGD.

Using the outcomes of Table 8 and Table 9, (see appendix)

$m_1 = 4,\ m_2 = 4,\ m^2 \times m^2 = 256,\ S^2{}_{RB} = 123.44, S^2{}_{cB} = 106.666 +\ E_2 = 15.1018,\ E_1 = 18.982$

Comparative Efficiency $= E_2 \,/\, E_1 = 0.701$

7. RESULTS AND DISCUSSION

As in Table 8 and in Table 9 the mean of error sum of squares of Hyper-Graeco-Latin sudoku square design (HGD) and Hyper-Block GLaSS design (HBGD) for the same hypothetical data as shown in Table 8 have been calculated.

To compare the two designs HGD and HBGD i.e., by removing the variability due to Row and column blocking in our proposed design i.e., HBGD has decreased the experimental error. In HGD error mean square is 18.98 and error mean square of HBGD is 15.10 which is less than the error mean square of HGD for the same hypothetical data. So it is effective or favorable to use row blocking and column blocking. As HBGD possess the properties of both HGD and Sudoku Square design so the result of HBGD are authentic because it gives less mean square for error.

CONCLUSION

Hyper Block Graeco Latin Sudoku Square Design (Hyper Block GLaSS Design) is the more efficient design as the row blocking and column blocking introduced in the Hyper Graeco Latin Sudoku Square Design. Introducing the Row Block Sum of Square and column Block sum of square in the new proposed design, the error sum of square is further reduced. The purpose of Hyper Block GLaSS design (HBGD) is to test three sets of treatments simultaneously in one experiment and allows investigation of eight factors. Parameter estimation with Random effect model and mixed effects model with ANOVA is given in this paper. The efficiency of the new proposed design is checked through numerical example by using hypothetical data set has come to the conclusion that the new proposed design is more efficient than Hyper Graeco Latin Soduko Square Design because Mean Square Error of Hyper Block GLaSS Design is minimum than HGD. Further, with the help of relative efficiency of the two designs shows that HBGD is more efficient than HGD. Hence the supplementary blocking factor makes the result more reliable and with the use HBGD one can control variability from eight sides with less error mean square

Appendix

Table 1: $m^2 \times m^2$ Hyper GLaSS Design for any values of m (odd, even)

		Column Block I			Column Block II			Column Block III		
		1	2	3	4	5	6	7	8	9
Rows Block 1	1	$G_{1,1,m^2}$	$G_{m+1, 2,2m}$	$G_{2m+1,m,m}$		$G_{m+2, m+2,m+2}$	$G_{2m+2, 2m,2}$	$G_{m,2m+1, 2m+1}$	$G_{2m,2m+2, m+1}$	$G_{m^2, m^2,1}$

	2	$G_{2,m+1,1}$	$G_{m+2,m+2,2m+1}$	$G_{2m+2,2m,m+1}$	$G_{m,2m+1,m^2}$	$G_{2m,2m+2,2m}$	$G_{m^2,m^2,m}$	$G_{m+1,1,2m+2}$	$G_{2m+1,2,m+2}$	$G_{1,m,2}$
	3	$G_{m,2m+1,2}$	$G_{2m,2m+2,2m+2}$	$G_{m^2,m^2,m+2}$	$G_{m+1,1,1}$	$G_{2m+1,2,2m+1}$	$G_{1,m,m+1}$	$G_{m+2,m+1,m^2}$	$G_{2m+2,m+1,2m}$	$G_{2,2m,m}$
Rows Block II	4	$G_{m+1,2,m}$	G_{2m+1,m,m^2}	$G_{1,m+1,2m}$	$G_{m+2,m+2,2}$	$G_{2m+2,2m,2m+2}$	$G_{2,2m+1,m+2}$	$G_{2m,2m+2,1}$	$G_{m^2,m^2,2m+1}$	$G_{m,1,m+1}$
	5	$G_{m+2,m+2,m+1}$	$G_{2m+2,2m,1}$	$G_{2,2m+1,2m+1}$	$G_{2m,2m+2,m}$	G_{m^2,m^2,m^2}	$G_{m,1,2m}$	$G_{2m+1,2,2}$	$G_{1,m,2m+2}$	$G_{m+1,m+1,m+2}$
	6	$G_{2m,2m+2,m+2}$	$G_{m^2,m^2,2}$	$G_{m,1,2m+2}$	$G_{2m+1,2,m+1}$	$G_{1,m,1}$	$G_{m+1,m+1,2m+1}$	$G_{2m+2,m+2,m}$	$G_{2,2m,m^2}$	$G_{m+2,2m+1,2m}$
Rows Block III	7	$G_{2m+1,m,2m}$	$G_{1,m+1,m}$	$G_{m+1,m+2,m^2}$	$G_{2m+2,2m,m+2}$	$G_{2,2m+1,2}$	$G_{m+2,2m+2,2m+2}$	$G_{m^2,m^2,m+1}$	$G_{m,1,1}$	$G_{2m,2,2m+1}$
	8	$G_{2m+2,2m,2m+1}$	$G_{2,2m+1,m+1}$	$G_{m+2,2m+2,1}$	$G_{m^2,m^2,2m}$	$G_{m,1,m}$	$G_{2m,2,m^2}$	$G_{1,m,m+2}$	$G_{m+1,m+1,2}$	$G_{2m+1,m+2,2m+2}$
	9	$G_{m^2,m^2,2m+2}$	$G_{m,1,m+2}$	$G_{2m,2,2}$	$G_{1,m,2m+1}$	$G_{m+1,m+1,m+1}$	$G_{2m+1,m+2,1}$	$G_{2,2m,2m}$	$G_{m+2,2m+1,m}$	$G_{2m+2,2m+2,m^2}$

Note: G indicates Hyper Block GLaSS design, where the first subscripts represent treatments type (1), the second subscripts represent treatments type (2) and the third subscripts represent treatments type (3).

Table 2: Complete Sudoku square design of treatments type (1) with the initial row

{A, E, I, M, B, F, J, N, C, G, K, O, D, H, L and P }

	Column Block 1				Column Block 2				Column Block 3				Column Block4			
Row Block II	A	E	I	M	B	F	J	N	C	G	K	O	D	H	L	P
	B	F	J	N	C	G	K	O	4	H	L	P	E	I	M	A
	C	G	K	O	D	H	L	P	E	I	M	A	F	J	N	B
	D	H	L	P	E	I	M	A	F	J	N	B	G	K	O	C
	E	I	M	A	F	J	N	B	G	K	O	C	H	L	P	D
	F	J	N	B	G	K	O	C	H	L	P	D	I	M	A	E

Row Block II	G	K	O	C	H	L	P	D	I	M	A	E	J	N	B	F
	H	L	P	D	I	M	A	E	J	N	B	F	K	O	C	G
Row Block III	I	M	A	E	J	N	B	F	K	O	C	G	L	P	D	H
	J	N	B	F	K	O	C	G	L	P	D	H	M	A	E	I
	K	O	C	G	L	P	D	H	M	A	E	I	N	B	F	J
	L	P	D	H	M	A	E	I	N	B	F	J	O	C	G	K
Row Block IV	M	A	E	I	N	B	F	J	O	C	G	K	P	D	H	L
	N	B	F	J	O	C	G	K	P	D	H	L	A	E	I	M
	O	C	G	K	P	D	H	L	A	E	I	M	B	F	J	N
	P	D	H	L	A	E	I	M	B	F	J	N	C	G	K	O

Table 2 shows that the numbers in the first row of each of the sub squares generates the matrix of order 4 with numbers 1 to 16 appears only once. Then its orthogonal Sudoku square design of treatments type (2) with the initial row {A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, and P} is in Table 3 as;

Table 3: Orthogonal Sudoku square design of treatments type (2) with the initial row {A, B,C, D, E, F, G, H, I, J, K, L, M, N, O, and P}

	Column Block I				Column Block II				Column Block III				Column Block IV			
Row Block I	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
	E	F	G	H	I	J	K	L	M	N	O	P	A	B	C	D
	I	J	K	L	M	N	O	P	A	B	C	D	E	F	G	H
	M	N	O	P	A	B	C	D	E	F	G	H	I	J	K	L
Row Block II	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	A
	F	G	H	I	J	K	L	M	N	O	P	A	B	C	D	E
	J	K	L	M	N	O	P	A	B	C	D	E	F	G	H	I
	N	O	P	A	B	C	D	E	F	G	H	I	J	K	L	M
Row Block III	C	D	E	F	G	H	I	J	K	L	M	N	O	P	A	B
	G	H	I	J	K	L	M	N	O	P	A	B	C	D	E	F
	K	L	M	N	O	P	A	B	C	D	E	F	G	H	I	J
	O	P	A	B	C	D	E	F	G	H	I	J	K	L	M	N

Row Block IV	D	E	F	G	H	I	J	K	L	M	N	O	P	A	B	C
	H	I	J	K	L	M	N	O	P	A	B	C	D	E	F	G
	L	M	N	O	P	A	B	C	D	E	F	G	H	I	J	K
	P	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O

Table 3 shows that the numbers in the first row of each of the sub squares generates the matrix of order 4 with numbers 1 to 16 appears only once. Then its orthogonal Sudoku square design of treatments type (3) with the initial row {P, L, H, D, O, K, G, C, N, J, F, B, M, I, E, A} is in Table 4 as

Table 4: Orthogonal Sudoku square design of treatments type (3) of order $(4x4)^2$

	Column Block I				Column Block II				Column Block III				Column Block IV			
Row Block I	p	L	H	D	O	K	G	C	N	J	F	B	M	I	E	A
	A	M	I	E	P	L	H	D	0	K	G	C	N	J	F	B
	B	N	J	F	A	M	I	E	P	L	H	d	O	K	G	C
	C	O	K	G	B	N	J	F	A	M	I	E	P	L	H	D
Row Block II	d	P	L	H	C	O	K	G	B	N	J	F	A	M	I	E
	E	A	M	I	D	P	L	H	C	O	K	G	B	N	J	f
	F	B	N	J	E	A	M	I	E	P	L	H	C	O	K	G
	G	C	O	K	F	B	N	J	F	A	M	I	D	P	L	H
Row Block III	H	D	P	L	G	C	O	K	G	B	N	J	E	A	M	I
	I	E	A	M	H	D	P	L	H	C	O	K	F	B	N	J
	J	f	B	N	I	E	A	M	I	D	P	L	G	C	O	K
	K	G	C	O	J	G	B	N	J	E	A	M	H	D	P	L
Row Block IV	L	H	D	P	K	H	C	O	K	F	B	N	I	E	A	M
	M	I	E	A	L	I	D	p	L	G	C	O	J	F	B	N
	N	J	F	B	M	J	E	1	M	H	D	P	K	G	C	O
	O	K	G	C	N	K	F	2	N	I	E	a	L	H	D	P

By super imposing the three types of Sudoku square designs given in Table 2, Table 3 and Table 4, we get the $(4x4)^2$ Hyper Block GLaSS design given in Table 5.

Table 5: Layout of Hyper Block GLaSS Design $(4 \times 4)^2$, =16*16=256 (m=4)

	Column Block I				**Column Block II**				**Column Block III**				**Column IV**			
Row Block I	**AAP**	**EBL**	**ICH**	**MDD**	**BEO**	**FFK**	**JGG**	**NHC**	**CIN**	**GJJ**	**KKF**	**OLB**	**DMM**	**HNI**	**LOO**	**PPA**
	BEA	**FFM**	**JGI**	**NHE**	**CIP**	**GJL**	**KKH**	**OLD**	**DMO**	**HNK**	**LOG**	**PPC**	**EAN**	**IBJ**	**MCF**	**ADB**
	CIB	**GJN**	**KKJ**	**OLF**	**DMA**	**HNM**	**LOI**	**PPE**	**EAP**	**IBL**	**MCH**	**ADD**	**FEO**	**JFK**	**NGG**	**BHC**
	DMC	**HNO**	**LOK**	**PPG**	**EAB**	**IBN**	**MCJ**	**ADF**	**FEA**	**JFM**	**NGI**	**BHE**	**GIP**	**KJL**	**OKH**	**CLD**
Row Block II	**EBD**	**ICP**	**MDL**	**AEH**	**FFC**	**JGO**	**NHK**	**BIG**	**GJB**	**KKN**	**OLJ**	**CMF**	**HNA**	**LOM**	**PPI**	**DAE**
	FFE	**JGA**	**NHM**	**BII**	**GJD**	**KKP**	**OLL**	**CMH**	**HNC**	**LOO**	**PPK**	**DAG**	**IBB**	**MCN**	**ADJ**	**EEF**
	GJF	**KKL**	**OLN**	**CMJ**	**HNE**	**LOA**	**PPM**	**DAI**	**IBD**	**MCP**	**ADL**	**EEH**	**JFC**	**NGO**	**BHK**	**FIG**
	HNG	**LOC**	**PPO**	**DAK**	**IBF**	**MCB**	**ADN**	**EEJ**	**JFE**	**NGA**	**BHM**	**FII**	**KJD**	**OKP**	**CLL**	**GMH**
Row Block III	**ICH**	**MDD**	**AEP**	**EFL**	**JGG**	**NHC**	**BIO**	**FJK**	**KKF**	**OLB**	**CMN**	**GNJ**	**LOE**	**PPA**	**DAM**	**HBI**
	JGI	**NHE**	**BIA**	**FJM**	**KKH**	**OLD**	**CMP**	**GNL**	**LOG**	**PPC**	**DAO**	**HBK**	**MCF**	**ADB**	**EEN**	**IFJ**

	KKJ	**OLF**	**CMB**	**GNN**	**LOI**	**PPE**	**DAA**	**HBM**	**MCH**	**ADD**	**EEP**	**IFL**	**NGG**	**BHC**	**FIO**	**JJK**
	LOK	**PPG**	**DAC**	**HBO**	**MCJ**	**ADF**	**EEB**	**IFN**	**NGI**	**BHE**	**FIA**	**JJM**	**OKH**	**CLD**	**GMP**	**KNL**
Row Block IV	**MDL**	**AEH**	**EFD**	**IGP**	**NHK**	**BIG**	**FJC**	**JKO**	**OLJ**	**CMF**	**GNB**	**KON**	**PPI**	**DAE**	**HBA**	**LCM**
	NHM	**BII**	**FJE**	**JKA**	**OLL**	**CMH**	**GND**	**KOP**	**PPK**	**DAG**	**HBC**	**LCP**	**ADJ**	**EEF**	**IFB**	**MGN**
	OLN	**CMJ**	**GNF**	**KOB**	**PPM**	**DAI**	**HBE**	**LCA**	**ADL**	**EEH**	**IFD**	**MGP**	**BHK**	**FIG**	**JJC**	**NKO**
	PPO	**DAK**	**HBG**	**LCC**	**ADN**	**EEJ**	**IFF**	**MGB**	**BHM**	**FII**	**JJE**	**NKA**	**CLL**	**GMH**	**KND**	**OOP**

Table 6: ANOVA Table for $m^2 \times m^2$ Hyper Block GLaSS Design

Sources of variation	d.f	SS	MS	E(SS)	E(MS)
Rows	$m^2 - 1$	$SSR = \frac{1}{m^2}\sum_{k=1}^{m^2} R^2_{k.(......)} - C.F.$	$S^2_R = \frac{SSR}{m^2 - 1}$	$m^2(m^2-1)\sigma^2_\alpha + (m^2-1)\sigma^2$	$m^2\sigma^2_\alpha + \sigma^2$
Columns	$m^2 - 1$	$SSC = \frac{1}{m^2}\sum_{l=1}^{m^2} C^2_{.l(......)} - C.F.$	$S^2_C = \frac{SSC}{m^2 - 1}$	$m^2(m^2-1)\sigma^2_\beta + (m^2-1)\sigma^2$	$m^2\sigma^2_\beta + \sigma^2$
Treatments type(A)	$m^2 - 1$	$SST(\text{A}) = \frac{1}{m^2}\sum_{r=1}^{m^2} T^2_{..(r.....)} - C.F.$	$S^2_{T(1)} = \frac{SST}{m^2 - 1}$	$m^2(m^2-1)\sigma^2_\tau + (m^2-1)\sigma^2$	$m^2\sigma^2_\tau + \sigma^2$
Blocks	$m^2 - 1$	$SSB = \frac{1}{m^2}\sum_{l=1}^{m^2} B^2_{..(.p....)} - C.F.$	$S^2 B = \frac{SSR}{m^2 - 1}$	$m^2(m^2-1)\sigma^2_\gamma + (m^2-1)\sigma^2$	$m^2\sigma^2_\gamma + \sigma^2$

Treatments type(B)	m^2-1	$SST(\text{B}) = \frac{1}{m^2}\sum_{q=1}^{m^2} T^2_{..(..q...)} - C.F.$	$S^2T_{(2)} = \frac{SST(2)}{m^2-1}$	$m^2(m^2-1)\sigma^2_\theta + (m^2-1)\sigma^2$	$m^2\sigma^2_\theta + \sigma^2$
Treatments type(C)	m^2-1	$SST(\text{C}) = \frac{1}{m^2}\sum_{s=1}^{m^2} T^2_{..(...s..)} - C.F.$	$S^2T_{(3)} = \frac{SST(3)}{m^2-1}$	$m^2(m^2-1)\sigma^2_\lambda + (m^2-1)\sigma^2$	$m^2\sigma^2_\lambda + \sigma^2$
Row Blocks	m-1	$SSRB = \frac{1}{m^3}\sum_{t=1}^{m} RB^2_{..(....t.)} - C.F.$	$S^2RB = \frac{SSRB}{m-1}$	$m^3(m-1)\sigma^2\varphi + (m-1)\sigma^2$	$m^3\sigma^2_\varphi + \sigma^2$
Column Block	m-1	$SSCB = \frac{1}{m^3}\sum_{u=1}^{m} CB^2_{..(.....u)} - C.F.$	$S^2CB = \frac{SSCB}{m-1}$	$m^3(m-1)\sigma^2\varphi + (m-1)\sigma^2$	$m^3\sigma^2\varphi + \sigma^2$
Error	$(m-1)\left[(m+1)(m^2-7)-2\right]$	By Subtraction	SSE/d.f	$\sigma^2(m-1)\left[(m+1)(m^2-7)-2\right]$	σ^2
Total	m^4-1	$TSS = \sum_{k=1}^{m^2}\sum_{l=1}^{m^2} y^2_{kl(rpqstu)} - C.F.$			

Table 7: Hypothetical data for Hyper Block GLaSS Design of order 16

		Column I				Column II				Column III				Column IV			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Row Block I	1	A_1, A_2, P_3 (20)	E_1, B_2, L_3 (25)	I_1, C_2, H_3 (29)	M_1, D_2, D_3 (25)	B_1, E_2, O_3 (23)	F_1,F_2,K_3 (28)	J_1,G_2,G_3 (24)	N_1, H_2, C_3 (19)	C_1,I_2,N_3 (21)	G_1,J_2,J_3 (16)	K_1, K_2, F_3 (10)	O_1, L_2, B_3 (14)	D_1, M_2, M_3 (16)	H_1, N_2,I_3 (20)	L_1, O_2, E_3 (15)	P_1, P_2, A_3 (10)
	2	B_1, E_2, A_3 (15)	F_1, F_2, M_3 (21)	J_1, G_2, I_3 (17)	N_1, H_2, E_3 (20)	C_1,I_2,P_3 (21)	G_1,J_2,L_3 (23)	K_1, K_2, H_3 (18)	O_1, L_2, D_3 (15)	D_1, M_1, O_3 (16)	H_1, N_2, K_3 (21)	L_1, O_2, G_3 (15)	P_1,P_2,C_3 (19)	E_1, A_2, N_3 (21)	I_1,B_2,J_3 (26)	M_1, C_2, F_3 (20)	A_1, D_2, B_3 (25)

	3	C_1, I_2, B_3 (19)	G_1, J_2, N_3 (26)	K_1, K_2, J_3 (22)	$O_1 L_2 F_3$ (24)	D_1, M_2, A_3 (23)	H_1, N_2, M_3 (17)	L_1, O_2, I_3 (25)	P_1, P_2, E_3 (20)	E_1, A_2, P_3 (21)	I_1, B_1, L_3 (25)	M_1, C_2, H_3 (20)	A_1, D_2, D_3 (23)	F_1, E_2, O_3 (22)	J_1, F_2, K_3 (18)	N_1, G_2, G_3 (19)	B_1, H_2, C_3 (29)
	4	D_1, M_2, C_3 (26)	H_1, N_2, O_3 (20)	L_1, O_2, K_3 (15)	P_1, P_2, G_3 (10)	E_1, A_2, B_3 (13)	I_1, B_2, N_3 (22)	M_1, G_2, J_3 (18)	A_1, D_2, F_3 (15)	F_1, E_2, A_3 (16)	J_1, F_2, M_3 (19)	N_1, G_2, I_3 (24)	B_1, H_2, E_3 (18)	G_1, I_2, P_3 (20)	K_1, J_2, L_3 (24)	O_1, K_2, H_3 (29)	C_1, L_2, D_3 (33)
Row Block II	5	F_1, B_2, D_3 (28)	I_1, C_2, P_3 (19)	M_1, D_2, L_3 (14)	A_1, E_2, H_3 (9)	F_1, F_2, C_3 (12)	J_1, G_2, O_3 (21)	N_1, H_2, K_3 (16)	B_1, I_2, G_3 (10)	G_1, J_2, B_3 (18)	K_1, K_2, N_3 (22)	O_1, L_2, J_3 (19)	C_1, M_2, F_3 (22)	H_1, N_2, A_3 (21)	L_1, O_2, M_3 (23)	P_1, P_2, I_3 (27)	D_1, A_2, E_3 (30)
	6	F_1, F_2, E_3 (24)	J_1, G_2, A_3 (15)	N_1, H_2, M_3 (20)	B_1, I_2, I_3 (15)	G_1, J_2, D_3 (16)	K_1, K_2, P_3 (24)	O_1, L_2, L_3 (11)	C_1, M_2, H_3 (16)	H_1, N_2, C_3 (14)	L_1, O_2, O_3 (27)	P_1, P_2, K_3 (24)	D_1, A_2, G_3 (27)	I_1, B_2, B_3 (26)	M_1, C_2, N_3 (19)	A_1, D_2, J_3 (23)	E_1, E_2, F_3 (26)
	7	G_1, J_2, F_3 (27)	K_1, K_2, B_3 (24)	O_1, L_2, N_3 (17)	C_1, M_2, J_3 (20)	H_1, N_2, E_3 (22)	L_1, O_2, A_3 (19)	P_1, P_2, M_3 (19)	D_1, A_2, I_3 (20)	I_1, B_2, D_3 (19)	M_1, C_2, P_3 (24)	A_1, D_2, L_3 (19)	E_1, E_2, H_3 (33)	$J_1 F_2 C_3$ (30)	N_1, G_2, O_3 (25)	B_1, H_2, K_3 (19)	F_1, I_2, G_3 (21)
	8	H_1, N_2, G_3 (23)	L_1, O_2, C_3 (17)	P_1, P_2, O_3 (22)	D_1, A_2, K_3 (25)	I_1, B_2, F_3 (28)	M_1, C_2, B_3 (23)	A_1, D_2, N_3 (25)	E_1, E_2, J_3 (18)	J_1, F_2, E_3 (16)	N_1, G_2, A_3 (19)	B_1, H_2, M_3 (25)	F_1, I_2, I_3 (29)	$K_1 J_2 D_3$ (27)	O_1, K_2, P_3 (20)	C_1, L_2, L_3 (24)	G_1, M_2, H_3 (19)
Row Block III	9	I_1, C_2, H_3 (22)	M_1, D_2, D_3 (19)	A_1, E_2, P_3 (25)	E_1, F_2, L_3 (22)	J_1, G_2, G_3 (25)	N_1, H_2, C_3 (20)	B_1, I_2, O_3 (23)	F_1, J_2, K_3 (20)	K_1, K_2, F_3 (19)	O_1, L_2, B_3 (24)	C_1, M_2, N_3 (18)	J_1, N_2, J_3 (27)	$L_1 O_2 E_3$ (26)	P_1, P_2, A_3 (19)	D_1, A_2, M_3 (25)	H_1, B_2, I_3 (20)
	10	J_1, G_2, I_3 (18)	N_1, H_2, E_3 (14)	B_1, I_2, A_3 (19)	F_1, J_2, M_3 (15)	K_1, K_2, H_3 (17)	O_1, L_2, D_3 (24)	C_1, M_2, P_3 (28)	G_1, N_2, L_3 (24)	L_1, O_2, G_3 (23)	P_1, P_2, C_3 (29)	D_1, A_2, O_3 (22)	H_1, B_2, K_3 (21)	$M_1 C_2 F_3$ (22)	A_1, D_2, B_3 (24)	E_1, E_2, N_3 (20)	I_1, F_2, J_3 (16)
	11	K_1, K_2, J_3 (23)	O_1, L_2, F_3 (20)	C_1, M_2, B_3 (15)	G_1, N_2, N_3 (20)	L_1, O_2, I_3 (16)	P_1, P_2, E_3 (20)	D_1, A_2, A_3 (25)	H_1, B_2, M_3 (20)	M_1, C_2, H_3 (24)	A_1, D_2, D_3 (23)	E_1, E_2, P_3 (19)	I_1, F_2, L_3 (15)	N_1, G_2, G_3 (17)	B_1, H_2, C_3 (20)	F_1, I_2, O_3 (15)	J_1, J_2, K_3 (21)

	12	L_1, O_2, K_3 (19)	P_1, P_2, G_3 (25)	D_1, A_2, C_3 (26)	H_1, B_2, O_3 (17)	M_1, C_2, J_3 (19)	A_1, D_2, F_3 (25)	E_1, E_2, B_3 (19)	I_1, F_2, N_3 (25)	N_1, G_2, I_3 (20)	B_1, H_2, E_3 (27)	F_1, I_2, A_3 (23)	J_1, J_2, M_3 (10)	O_1, K_2, H_3 (11)	C_1, L_2, D_3 (15)	G_1, M_2, P_3 (10)	K_1, N_2, L_3 (15)
Row Block IV	13	M_1, D_2, L_3 (22)	A_1, E_2, H_3 (24)	E_1, F_2, D_3 (27)	I_1, G_2, P_3 (20)	N_1, H_2, K_3 (21)	B_1, I_2, G_3 (26)	F_1, J_2, C_3 (21)	J_1, K_2, O_3 (26)	O_1, L_2, J_3 (24)	C_1, M_2, F_3 (26)	G_1, N_2, B_3 (22)	K_1, O_2, N_3 (12)	P_1, P_2, I_3 (13)	D_1, A_2, E_3 (17)	H_1, B_2, A_3 (11)	L_1, C_2, M_3 (17)
	14	N_1, H_2, M_3 (26)	B_1, I_2, I_3 (28)	F_1, J_2, E_3 (22)	J_1, K_2, A_3 (16)	O_1, L_2, L_3 (17)	C_1, M_2, H_3 (22)	G_1, N_2, D_3 (26)	K_1, O_2, P_3 (20)	P_1, P_1, K_3 (19)	D_1, A_2, G_3 (22)	H_1, B_2, C_3 (27)	L_1, C_2, O_3 (17)	A_1, D_2, J_3 (18)	E_1, E_2, F_3 (22)	I_1, F_2, B_3 (17)	M_1, G_2, N_3 (22)
	15	O_1, L_2, N_3 (21)	C_1, M_2, J_3 (25)	G_1, N_2, F_3 (29)	K_1, O_2, B_3 (33)	P_1, P_2, M_3 (32)	D_1, A_2, I_3 (27)	H_1, B_2, E_3 (31)	L_1, C_2, A_3 (25)	A_1, D_2, L_3 (23)	E_1, E_2, H_3 (27)	I_1, F_2, D_3 (33)	M_1, G_2, P_3 (25)	B_1, H_2, K_3 (24)	F_1, I_2, G_3 (27)	J_1, J_2, C_3 (24)	N_1, K_2, O_3 (19)
	16	P_1, P_2, O_3 (16)	D_1, A_2, K_3 (20)	H_1, B_2, G_3 (25)	L_1, C_2, C_3 (29)	A_1, D_2, N_3 (27)	E_1, E_2, J_3 (33)	I_1, F_2, F_3 (29)	M_1, G_2, B_3 (21)	B_1, H_2, M_3 (20)	F_1, I_2, I_3 (24)	J_1, J_2, E_3 (29)	N_1, K_2, A_3 (20)	C_1, L_2, L_3 (21)	G_1, M_2, H_3 (32)	K_1, N_2, D_3 (29)	O_1, O_2, P_3 (24)

Table 8: ANOVA Table for 16×16 Hyper Graeco-Latin Sudoku Square Design

Sources of Variation	d.f	SS	MS	F
Rows	15	997.6250	66.5083	3.5035
Columns	15	293.7501	19.5833	1.03162
Treatments type (1)	15	311	20.7330	1.0921
Blocks	15	913.6250	60.9083	3.2085
Treatments type (2)	15	196.5001	13.1003	0.69
Treatments type (3)	15	359.7503	23.9833	1.2634
Error	165	3132.1875	18.9829	

Total	255	6164		

Table 9: ANOVA Table for 16×16 Hyper Block GLaSS Design

Sources of Variation	d.f	SS	MS	F
Rows	15	997.6250	66.5083	3.5035
Columns	15	293.7501	19.5833	1.03162
Type (1) treatment	15	311	20.7330	1.0921
Blocks	15	913.6250	60.9083	3.2085
Type (2) Treatments	15	196.5001	13.1003	0.69
Type (3) Treatments	15	359.7503	23.9833	1.2634
Row Blocks	3	370.3281	123.4427	2.01
Column Blocks	3	320	1o6.666	1.320
Error	159	2401.1875	15.1018	
Total	255	6164		

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*As the observed values are less than the tabulated values with 5% significance level hence these are insignificant effects.