Analysis On Mathematical Inventory Control Policy Models For Deteriorating Products With Various Costs By Using Exponential Function

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ABSTRACT

In this paper, we estimate the control policies for deteriorating products with various costs by using exponential function. Deteriorating inventory models with exponential function of various costs with shortage are developed. Shortages are acceptable in a production process that is operating well. The fully backlogged portion balances this. Due to varying exponential costs, deterioration rate is also changeable with time function. Demand items are considered as a constant, which can be obeyed in the exponential function of various costs. We develop the stock keeping cost function as exponential with varying time. Based on these deductions, a mathematical model has been developed and solved numerically. Using the help of numerical analysis we obtain optimum order quantity for each cycle.

KEY WORDS

Exponential varying cost, Shortages, deteriorating rate, exponential function, Inventory Costs

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1. INTRODUCTION

Zhang and Wang [18] proposed a mathematical model for inventory management in supply chain networks that takes into account multiple demand scenarios, supply chain structure, and inventory policies. The desired outcome of the concept is to reduce the overall cost of inventory while maintaining a high standard of customer service. Zhu and Cui[21] developed a novel mathematical model for inventory control in a multi-level supply chain network. The model considers the coordination of inventory decisions between different echelons of the supply chain and aims to optimize the total inventory cost while satisfying customer demand. Li et al. [8] proposed a new mathematical model for inventory management in a two-tier supply chain that accounts for demand uncertainty and lead time variations. Xu et al. [16]proposed an improved mathematical model for inventory control in a closed-loop supply chain that takes into account the reverse logistics process. The model aims to optimize the total inventory cost while minimizing the environmental impact of the supply chain. Gao et al.[4] created a stochastic mathematical model for inventory management in a multi-echelon supply chain that accounts for both demand uncertainty and lead time unpredictability.

Chen et al. [2] proposed a mathematical model for inventory control in a dual-channel supply chain that takes into account the competition between direct and indirect channels. The aim of the model is to optimize inventory allocation across the two channels in order to maximize the supply chain's overall profit. Cheng et al. [3] developed a mathematical model for inventory management in a multi-echelon supply chain that considers the impact of supplier reliability on inventory decisions. Xie et al. [15] presented a new mathematical model for inventory management in a three-echelon supply chain with multiple retailers. The model considers demand uncertainty, lead time variability and stock out risk. Lee and Kim [6] proposed a mathematical model for inventory management in a supply chain with a single supplier and multiple retailers. The model considers demand uncertainty and lead time variability. Chen and Liu [1] proposed a mathematical model for inventory management in a two-echelon supply chain with demand uncertainty and lead time variability. The aim of the model is to optimize the inventory strategy in order to maximize the supply chain's expected profit.

Yang et al. [17] created a new mathematical model for inventory control in a supply chain that includes several suppliers and retailers. The model considers demand uncertainty and supply disruption risk. Su et al. [10] presented a mathematical model for inventory management in a supply chain with a single supplier and multiple retailers. The model considers demand uncertainty and lead time variability and aims to optimize the inventory policy to maximize the total profit of the supply chain. Zhang et al. [19] suggested a two-stage stochastic programming approach for inventory management in a multi-tier supply chain with several suppliers and retailers. The model considers demand uncertainty, lead time variability, and supply disruption risk. Sun et al. [11] created a mathematical model for inventory control in a supply chain that includes a single supplier and many retailers. The model takes into account demand unpredictability and lead time variations, with the goal of optimizing the inventory strategy to maximize the supply chain's overall profit. Wang et al. [13] suggested a two-stage stochastic programming approach for managing inventories in a supply chain with various merchants . The model considers demand uncertainty and lead time variability and aims to minimize the total cost of inventory while ensuring a high level of service level for customers.

Li et al. [7] proposed a mathematical model for inventory management in a supply chain with demand uncertainty and supplier reliability. Wang et al.[14] developed a mathematical model for inventory control in a supply chain with a single supplier and multiple retailers. The model considers demand uncertainty and lead time variability and aims to optimize the inventory policy to maximize the total profit of the supply chain. Zhang, X., Xu, Y., & Wang, X. [20] created a mathematical model for joint pricing and varied inventory choices in a competitive supply chain that incorporates demand uncertainty. Guo, P et al. [5] proposed a mathematical model for control policy with free distribution demand and uncertainty demand consider. Wang, L et al. [12]developed a mathematical model for Coordinating inventory using pricing decisions in a dual channel supply chain with return policy. Liu, M., et al. [9] presented a mathematical model for supply chain coordination with various merchants when lead times are unpredictable. The model optimizes the inventory strategy to maximize the supply chain's overall profit while taking the influence of demand prediction accuracy into account.

2. Notations and Assumptions

Notations:

D- Constant demand and known,L_m is maximum life time and r, c_3 F - ordering cost, market price in dollars and rate of default risk respectively.

$h(m) = hk_1e^{-\alpha_1 L_m}$,$c_1(m) = c_1k_2e^{-\alpha_2 L_m}$,$k_1$,$\alpha_1 k_2$,$\alpha_2$ are positive constants.

$s(m) = sk_3e^{-\alpha_3 L_m}$, deterioration rate is $\theta(t) = \frac{1}{1+L_m-t}$, $0 \le t \le T$

Assumptions:

Shortages are allowed and backlogged them completely and the replenishment cycle time

$T\le L_m$ with $\theta(t) \le 1$

The following variables consider as decision-making variables

L_m^*(Optimal) maximum life time of products, T_1^*(optimal) the seller's shortage time in years and T^*(optimal) The seller's cycle time in years.

3. Formulation of Mathematical model

The inventory level at time t falls due to demand and deterioration. According to this description, the following differential equations may be used to represent inventory variations over time:;

$\frac{dI_1(t)}{dt} = -D - \theta(t)I_1(t), \quad 0 \le t \le T_1$(1)

$\frac{dI_2(t)}{dt} = -D, \quad T_1 \le t \le T$(2)

Using boundary condition $I_1(0) = 0, I_2(T) = 0$,

Hence the various inventory levels are

$I_1(t) = -D(1 + L_m - T_1)\log\left[\frac{1+L_m-t}{1+L_m-T_1}\right], 0 \le t \le T_1$ (3)

$I_2(t) = D[T - t], T_1 \le t \le T$ (4)

The maximum order quantity is

$Q_0 = I_1(0) + I_2(0)$

$Q_0 = D(1 + L_m - T_1)\log\left[\frac{1+L_m}{1+L_m-T_1}\right] + DT$(5)

(i) The average cost of ordering is $\frac{r}{T}$

(ii) Average stock holding cost is $\frac{h(m)}{T}\left\{\int_0^{T_1} I_1(t)dt\right\}$

$= \frac{hk_1e^{-\alpha_1 L_m}D}{T}\left\{(1 + L_m - T_1)\left[T_1 - (1 + L_m)\log\left[\frac{1+L_m-T_1}{1+L_m}\right]\right]\right\}$ (6)

(iii) Annual buying cost or selling price is $\frac{c_1(m)Q_0}{T}$

$= \frac{c_1k_2e^{-\alpha_2 L_m}D}{T}\left\{(1 + L_m - T_1)\log\left[\frac{1+L_m}{1+L_m-T_1}\right] + T\right\}$ (7)

(iv) Average shortage cost is $\frac{s(m)}{T}\left\{\int_{T_1}^{T} I_2(t)dt\right\}$

$= \frac{sk_3e^{-\alpha_3 L_m}D}{2T}(T - T_1)^2$ (8)

(v) Average yearly revenue following default risk is $Dc_3(1 - F)$ (9)

Hence, the seller's annual profit per unit time is given by

$$z(L_m, T_1, T) = Dc_3(1 - F) - \frac{r}{T} - \frac{h(m)}{T}\left\{\int_0^{T_1} I_1(t)dt\right\} - \frac{c_1(m)Q_0}{T} - \frac{s(m)D}{2T}\left\{\int_{T_1}^{T} I_2(t)dt\right\}$$

$$= Dc_3(1-F) - \frac{r}{T} - \frac{Dhk_1e^{-\alpha_1 L_m}}{T}\left\{(1+L_m-T_1)\left[T_1-(1+L_m)\log\left[\frac{1+L_m-T_1}{1+L_m}\right]\right]\right\} - \frac{k_2c_1e^{-\alpha_2 L_m}D}{T}\left\{(1+L_m-T_1)\log\left[\frac{1+L_m}{1+L_m-T_1}\right]+T\right\} - \frac{Dsk_3e^{-\alpha_3 L_m}}{2T}(T-T_1)^2$$

$$= D[c_3(1-F)-c_1k_2e^{-\alpha_2 L_m}] - \frac{r}{T} - \frac{D}{T}\left\{hk_1e^{-\alpha_1 L_m}(1+L_m-T_1)T_1 + (1+L_m-T_1)\log\left[\frac{1+L_m-T_1}{1+L_m}\right][hk_1(1+L_m)e^{-\alpha_1 L_m}+c_1k_2e^{-\alpha_2 L_m}] + \frac{sk_3e^{-\alpha_3 L_m}}{2}(T-T_1)^2\right\} \quad (10)$$

$L_m^*, (T_1^*, T^*)$ to maximize the seller's overall profit.

$$\frac{\partial z(L_m,T_1,T)}{\partial L_m} = 0, \frac{\partial z(L_m,T_1,T)}{\partial T_1} = 0, \frac{\partial z(L_m,T_1,T)}{\partial T} = 0 \qquad (11)$$

Provided the sufficient conditions $\frac{\partial^2 z(L_m,T_1,T)}{\partial {L_m}^2} \le 0, \frac{\partial^2 z(L_m,T_1,T)}{\partial {T_1}^2} \le 0, \frac{\partial^2 z(L_m,T_1,T)}{\partial T^2} \le 0.$

$$\frac{\partial z(L_m,T_1,T)}{\partial L_m} = 0$$

$$Dc_1k_2\alpha_2e^{-\alpha_2 L_m} - \frac{D}{T}\left\{-k_1\alpha_1e^{-\alpha_1 L_m}(1+L_m-T_1)T_1 + k_1e^{-\alpha_1 L_m}T_1 + (1+L_m-T_1)\log\left[\frac{1+L_m-T_1}{1+L_m}\right][-k_1\alpha_1(1+L_m)e^{-\alpha_1 L_m} - k_2\alpha_2e^{-\alpha_2 L_m} + k_1e^{-\alpha_1 L_m}] + \left(k_1(1+L_m)e^{-\alpha_1 L_m}+k_2e^{-\alpha_2 L_m}\right)\left(-\frac{1+L_m-T_1}{1+L_m}+\log\left[\frac{1+L_m-T_1}{1+L_m}\right]\right) - \frac{\alpha_3k_3e^{-\alpha_3 L_m}}{2}(T-T_1)^2\right\} = 0$$

$$\text{i.e.}, L_m = \frac{\frac{sD\alpha_3k_3(T-T_1)^2}{2} + hDk_1(1-T_1^2) + c_1Dk_2[T_1\alpha_2(T_1-1)-1] - TDc_1k_2\alpha_2}{hDk_1(\alpha_1T_1(1-2T_1)+\alpha_1-1-2T_1) + c_1Dk_2(T_1\alpha_2(1-\alpha_2+\alpha_2T_1)-T_1-\alpha_2) + \frac{sD\alpha_3^2k_3(T-T_1)^2}{2} - TDc_1k_2\alpha_3^2} \quad (12)$$

and $\frac{\partial z(L_m,T_1,T)}{\partial T_1} = 0$

$$\Rightarrow hk_1e^{-\alpha_1 L_m}(1+L_m-2T_1) - \left(hk_1(1+L_m)e^{-\alpha_1 L_m}+c_1k_2e^{-\alpha_2 L_m}\right)\left(1+\log\left[\frac{1+L_m-T_1}{1+L_m}\right]\right) - sk_3e^{-\alpha_3 L_m}(T-T_1) = 0$$

$$\text{i.e.}, T_1 = \frac{(1+L_m-\log(1+L_m))(hk_1(1+L_m)e^{-\alpha_1 L_m}+c_1k_2e^{-\alpha_2 L_m}) - hk_1(1+L_m)e^{-\alpha_1 L_m} + sk_3e^{-\alpha_3 L_m}T}{hk_1e^{-\alpha_1 L_m}(L_m-1) + c_1k_2e^{-\alpha_2 L_m} + sk_3e^{-\alpha_3 L_m}} \qquad (13)$$

and $\frac{\partial z(L_m,T_1,T)}{\partial T} = 0$

$$\frac{1}{T^2}\left\{r + D\left[hk_1e^{-\alpha_1 L_m}(1+L_m-2T_1)T_1 + (1+L_m-T_1)\log\left[\frac{1+L_m-T_1}{1+L_m}\right]\left(hk_1(1+L_m)e^{-\alpha_1 L_m}+c_1k_2e^{-\alpha_2 L_m}\right)\right]\right\} - sk_3e^{-\alpha_3 L_m}(T-T_1) = 0$$

$$, T = \frac{D(T_1-(1+L_m))\left\{hk_1 e^{-\alpha_1 L_m} T_1 + \log\left[\frac{1+L_m-T_1}{1+L_m}\right]\left(hk_1(1+L_m)e^{-\alpha_1 L_m} + c_1 k_2 e^{-\alpha_2 L_m}\right)\right\}}{Dsk_3 e^{-\alpha_3 L_m}} \quad (14)$$

By Equations (12), (13) and (14) we can obtain the optimal values $L_m^* = L_m$, $T_1^* = T_1$, $T^* = T$.

To find he optimal decision variables L_m^*,T_1^*,$T^*$to maximize the profit $z^*(L_m^*, T_1^*, T^*)$ by using the following algorithm.

Procedure to solve the problem

stage 1. Input the various values

stage2. Put the values into equation (12) and to find L_m

stage 3. Use $L_m$to obtain $T_1$by the equation (13)

stage4. Using $L_m, T_1$to determine T by the equation (14)

stage5. Set $L_m^* = L_m, T_1^* = T_1, T^* = T$

stage. Use the equation (10) to find $z^*(L_m^*, T_1^*, T^*)$

4. NUMERICAL EXAMPLE

D = 100 units,$k_1 = 2$, $k_2 = 1$, $k_3 = 3$, r =10, h = 0.2, s = 0.1, $c_1 = 0.5$, $c_3 = 4$, T = 0.2 year, $T_1 =$ 0.15 year, $\alpha_1 = 8$, $\alpha_2 = 4$, $\alpha_3 = 1$, F = 0.5

The optimal solutions are found as

$L_m^* = 0.7043$, $T^* = 0.6757$, $T_1^* = 0.2149$,$z^*(L_m^*, T_1^*, T^*) = 185.0366$

Table 1 Effect of changes in the various parameters

Decisions variables		$\mathbf{h(L_m^*)}$	$\mathbf{c_1(L_m^*)}$	$\mathbf{s(L_m^*)}$	$\mathbf{L_m^*}$	$\mathbf{T_1^*}$	$\mathbf{T^*}$	$\mathbf{z^*(L_m^*, T_1^*, T^*)}$
D	100	0.0014	0.0299	0.1483	0.7043	0.2149	0.6757	185.0366
	150	0.0014	0.0299	0.1483	0.7043	0.2149	0.4502	277.5250
	200	0.0014	0.0299	0.1483	0.7043	0.2149	0.3375	370.0149
	250	0.0014	0.0299	0.1483	0.7043	0.2149	0.2699	462.4998
r	5	0.0014	0.0299	0.1483	0.7043	0.2149	0.3386	185.0554
	6	0.0014	0.0299	0.1483	0.7043	0.2149	0.4060	185.0455
	7	0.0014	0.0299	0.1483	0.7043	0.2149	0.4734	185.0399
	8	0.0014	0.0299	0.1483	0.7043	0.2149	0.5408	185.0370
$\mathbf{c_3}$	6	0.0014	0.0299	0.1483	0.7043	0.2149	0.6757	285.0366
	8	0.0014	0.0299	0.1483	0.7043	0.2149	0.6757	385.0366

	10	0.001 4	0.0299	0.1483	0.704 3	0.214 9	0.675 7	485.0366
F	0.005	0.001 4	0.0299	0.1483	0.704 3	0.214 9	0.675 7	383.0366
	0.050	0.001 4	0.0299	0.1483	0.704 3	0.214 9	0.675 7	365.0366

Table2 Effect of changes in the various parameters

decision variables		$h(L_m^*)$	$c_1(L_m^*)$	$s(L_m^*)$	L_m^*	T_1^*	T^*	$z^*(L_m^*, T_1^*, T^*)$
α_1	8.0	0.001400	0.0299	0.1483	0.7043	0.2149	0.6757	185.0366
	8.5	0.000270	0.0161	0.1271	0.8588	0.2052	0.7886	187.2889
	9.0	0.000020	0.0061	0.0999	1.0999	0.2010	1.0033	190.0572
α_2	4.00	0.0014	0.0299	0.1483	0.7043	0.2149	0.6757	185.0366
	4.05	0.0020	0.0340	0.1544	0.6639	0.2182	0.6488	184.3699
	4.10	0.0026	0.0381	0.1601	0.6281	0.2217	0.6259	183.7450
α_3	0.52	0.0014	0.0299	0.2080	0.7045	0.2107	0.4825	179.1025
	0.54	0.0014	0.0299	0.2051	0.7045	0.2108	0.4893	179.3914
	0.56	0.0014	0.0299	0.2022	0.7045	0.2110	0.4962	179.6764

5. CONCLUSION

In this paper, we estimated the control policies for deteriorating products with various costs by using exponential function. This model developed on deteriorating inventory products with exponential function of various costs and shortages. Shortages are acceptable in a productive process. Full backlog serves as a counterbalance to this. Deterioration rate varies with time function due to different exponential costs. Demand components are thought of as a constant that can be followed in the exponential function of different expenses. We developed the stock keeping cost function as exponential with varying time. Based on these deductions, a mathematical model has been developed and solved numerically, which gives win-win out comes for both buyer's. Using the help of numerical analysis we obtained optimum order quantity for each cycle.

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