An Efficient Modified Ratio Estimator under Stratified Ranked Set Sampling

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Abstract: Stratified Ranked Set Sampling (SRSS) merges the benefits of stratification and Ranked Set Sampling (RSS) to yield an unbiased estimate of the population mean, with potential improvements in efficiency. This paper introduces modified ratio estimators for determining the mean of a finite population, using the first and third quartiles as supplementary information within the SRSS design. Results indicate that these estimators perform better compared to those based on Stratified Simple Random Sampling (StSRS). Expressions for bias and mean squared error (MSE) are derived for the proposed estimators. Theoretical analysis suggests that, under certain conditions, these estimators are more efficient than those used in StSRS and some existing SRSS methods.

Keywords: efficiency, population mean, ratio estimators, stratified ranked set sampling, conditions.

1. Introduction

Ranked Set Sampling (RSS) is a powerful statistical sampling technique introduced by McIntyre (1952) to improve the efficiency of estimators, particularly in scenarios where measurement is expensive or time-consuming, but ranking of units is relatively easy and inexpensive. Unlike Simple Random Sampling (SRS), RSS incorporates prior information about the population through ranking, enabling more informative and cost-effective sampling.

In RSS, multiple sets of units are randomly selected from the population, and within each set, units are ranked based on expert judgment, visual inspection, or some auxiliary variable. A representative unit from each ranked position is then measured, resulting in a sample that better reflects the distribution of the population. This method has been shown to produce more efficient estimators of population parameters such as the mean, variance, and proportion, compared to traditional sampling methods.

In ranked set sampling (RSS) the researcher first selects m^2 units from a specified population and these units are then distributed into m sets each of size m. Then, the researcher ranked the units within each set with some expert judgment. The smallest judged unit from the first set, the second smallest judged unit form the second set, the process continues until largest unit selected from the last set. Finally, m samples are selected for actual observation (Mclyntyre, 1951). In order to get more efficient estimators, and to make the RSS more applicable, it is modified by many researchers (Khan et al., 2020; Khan et al., 2022; Khan, Ismail and Noor-ul-Amin, 2022).

Researcher used various sampling method for selection of representative samples from population (Haq et al., 2024; Khan et al., 2024; Khan et al., 2024a; Cheema et al., 2024; Khan, Haq and Ali, 2022). While, the RSS method is better for costly survey/experiment to select the representative sample form the population Khan, Asadullah and Ali (2022). The RSS method is extended to stratified RSS by Samawi (1996). In StRSS, the population of size N is divided into L mutually exclusive and collectively exhaustive group (strata) of N_1, N_2, N_3,…..N_L units, such that $N_1+ N_2 +N_3+….+N_L=N$. Then, a sample is drawn from each stratum by using RSS technique. The sample size within the strata are denoted by n_1, n_2 ,n_3….n_L respectively. Sample size is as follows, $n=\sum_{h=1}^{L} n_h =\sum_{h=1}^{L} rm_h=rm$, where, “r” is number

of cycles, m_h is number of selected units in each stratum and "m" is number of selected units in each cycle.
The efficiency of estimator can be further improv by utilizing the auxiliary information. The variable of auxiliary information is called bench mark variable or auxiliary variable. While, the variable under consideration is referred to variable under study or variable of interest. For example, a researcher is interested in the total wheat production of the specific area for a given year. He may take the area under cultivation which is positively correlated to wheat production as auxiliary variable. The auxiliary variable must be correlated to variable of interest, if it is used for improvement in efficiency of the estimator. The ratio estimator is suitable method of estimation if the variable of interested and auxiliary variable has positive correlation (Cochran, 1977).
The ratio estimator under StRSS is investigated by Samawi and Saim (2003). The proposed estimator is then compared with existed estimators based on stratified simple random sampling (StSRS). Another ratio estimator under StRSS is proposed by Mandowara and Mehta (2014). Co-efficient of variation and coefficient of kurtosis are used as auxiliary variables in their proposed ratio estimator. Further, the proposed estimator is compared with some existent estimators. Khan, Shabbir and Gupta (2016) produced ratio estimators under ranked set sampling by using coefficient of correlation, coefficient of kurtosis and quartiles as auxiliary variable. Khan, Shabbir and Kalider (2016) presented ratio estimators under stratified ranked set sampling. They used coefficient of correlation, coefficient of variation and coefficient of kurtosis as auxiliary variables. Khan and Shabbir (2016a) presented ratio estimators under stratified double ranked set sampling. They used coefficient of variation and coefficient of kurtosis as auxiliary variables. Saini and Kumar (2019) introduced ratio estimators under stratified ranked set sampling. They use first and third quartiles as auxiliary variables in their proposed ratio estimators. Further they compared the proposed estimator with some existed estimators. Ahmad and Shabbir (2019) introduced new technique in StRSS, called stratified extreme-cum-median ranked set sampling. They proposed estimator by procedure which is the combination of stratified extreme ranked set sampling (StrERSS) and stratified median ranked set sampling (StrMRSS) called stratified extreme median ranked set sampling (StEMRSS). Further they compared the proposed estimator with existing estimator based on (StERSS) and (StMRSS). Khan and Ismail (2019) introduced ratio estimator under RSS using the known population parameter of auxiliary variable. The estimator performs better than its counterpart estimator subject to some conditions. Cetin and Koyuncu (2020) provided ratio estimators under stratified ranked set sampling. They used coefficient of variation and coefficient of kurtosis as auxiliary variables. Bhushan et al. (2022) introduced combined and separate difference and ratio type estimators of population mean under StRSS. Their study identified several well-known estimators as the sub-class of the proposed estimator.

2. Some Existing Estimators

Mandowara and Mehta (2014) suggested few ratio estimators under StRSS. They used population coefficient of variation in the first estimator, coefficient of kurtosis in the second estimator as auxiliary variable. Whereas in the third estimator they used both coefficient of variation and kurtosis as auxiliary variable. And in the fourth estimator, they used coefficient of kurtosis and coefficient of variation as auxiliary variable. The estimator is as follows;

$$\bar{y}_{strMM1} = \bar{y}_{(StRSS)} \left(\frac{\sum_{h=1}^{L} W_h (\bar{X}_h + C_{x_h})}{\sum_{h=1}^{L} W_h (\bar{x}_{h(rss)} + C_{x_h})} \right)$$

The Bais and MSE of the estimator is as follows,

$$Bias\ (\bar{y}_{strMM1}) = \bar{Y} \left[\sum_{h=1}^{L} \frac{W_h^{\ 2}}{n_h} \left\{ \frac{\lambda^2{}_1 S^2{}_{x_h}}{\bar{X}^2} - \frac{\lambda_1 S_{x_h y_h}}{\bar{X}\bar{Y}} \right\} - \sum_{h=1}^{L} \frac{W_h^{\ 2}}{n_h} \left\{ \frac{m}{n_h} \left(\lambda^2{}_1 \sum_{i=1}^{r_h} (D_{x_{h(i)}}{}^2 - \lambda_1 \sum_{i=1}^{r_h} D_{x_{h(i)} y_{h(i)}} \right) \right\} \right]$$

$$MSE(\bar{y}_{strMM1}) = \sum_{h=1}^{L} \frac{W_h^{\ 2}}{n_h} \left[(S^2{}_{y_h} + R^2 \lambda^2{}_1 S^2{}_{x_h} - 2R\lambda_1 S_{x_h y_h}) - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_{h(i)}} - \lambda_1 D_{x_{h(i)}})^2 \right]$$

Where, $W_h = \frac{N_h}{N}$, N_h is the n^{th} stratum size, N is the total population size. $\mu_{X(i)}$ and $\mu_{Y(i)}$ is the mean of i^{th} order statistics for X and Y respectively. $D_{x_{h(i)}} = (\mu_{xh(i)} - \bar{X}_h)$, $D_{y_{h(i)}} = (\mu_{yh(i)} - \bar{Y}_h)$, $D_{x_{h(i)}y_{h(i)}} = (\mu_{X(i)} - \bar{X}_h)(\mu_{Y(i)} - \bar{Y}_h)$.

Similarly, the second estimator is as follows,

$$\bar{y}_{strMM2} = \bar{y}_{(StRSS)} \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h + \beta_{2h}(x) \right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_{h(rss)} + \beta_{2h}(x) \right)} \right)$$

The bias and MSE of the proposed estimator is derived as below;

$$Bias\ (\bar{y}_{strMM2}) = \bar{Y} \left[\sum_{h=1}^{L} \frac{W_h^2}{n_h} \left\{ \frac{\lambda^2{}_2 S^2{}_{x_h}}{\bar{X}^2} - \frac{\lambda_1 S_{x_h y_h}}{\bar{X}\bar{Y}} \right\} - \sum_{h=1}^{L} \frac{W_h^2}{n_h} \left\{ \frac{m}{n_h} \left(\lambda^2{}_2 \sum_{i=1}^{r_h} (D_{x_{h(i)}}{}^2 - \lambda_2 \sum_{i=1}^{r_h} D_{x_{h(i)} y_{h(i)}} \right) \right\} \right]$$

$$MSE(\bar{y}_{strMM2}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h} \left[(S^2{}_{y_h} + R^2 \lambda^2{}_2 S^2{}_{x_h} - 2R\lambda_2 S_{x_h y_h}) - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_{h(i)}} - \lambda_2 D_{x_{h(i)}})^2 \right]$$

The third estimators is as under,

$$\bar{y}_{strMM3} = \bar{y}_{(StRSS)} \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h \beta_{2h}(x) + C_{x_h} \right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_{h(rss)} \beta_{2h}(x) + C_{x_h} \right)} \right)$$

The bias and MSE is derived as below;

$$Bias\ (\bar{y}_{strMM3}) = \bar{Y} \left[\sum_{h=1}^{L} \frac{W_h^2}{n_h} \left\{ \frac{\gamma_1{}^2 S^2{}_{x_h}}{\bar{X}^2} - \frac{\gamma_1 S_{x_h y_h}}{\bar{X}\bar{Y}} \right\} - \sum_{h=1}^{L} \frac{W_h^2}{n_h} \left\{ \frac{m}{n_h} \left(\gamma_1{}^2 \sum_{i=1}^{r_h} (D_{x_{h(i)}}{}^2 - \gamma_1 \sum_{i=1}^{r_h} D_{x_{h(i)} y_{h(i)}} \right) \right\} \right]$$

$$MSE(\bar{y}_{strMM3}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h} \left[(S^2{}_{y_h} + R^2 \gamma_1{}^2 S^2{}_{x_h} - 2R\gamma_1 S_{x_h y_h}) - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_{h(i)}} - \gamma_1 D_{x_{h(i)}})^2 \right]$$

The fourth estimator is as follows,

$$\bar{y}_{strMM4} = \bar{y}_{(StRSS)} \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h C_{x_h} + \beta_{2h}(x) \right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_{h(rss)} C_{x_h} + \beta_{2h}(x) \right)} \right)$$

The bias and MSE of this estimator is derived as below;

$$Bias\ (\bar{y}_{strMM4}) = \bar{Y} \left[\sum_{h=1}^{L} \frac{W_h^2}{n_h} \left\{ \frac{\gamma_2{}^2 S^2{}_{x_h}}{\bar{X}^2} - \frac{\gamma_2 S_{x_h y_h}}{\bar{X}\bar{Y}} \right\} - \sum_{h=1}^{L} \frac{W_h^2}{n_h} \left\{ \frac{m}{n_h} \left(\gamma_2{}^2 \sum_{i=1}^{r_h} (D_{x_{h(i)}}{}^2 - \gamma_2 \sum_{i=1}^{r_h} D_{x_{h(i)} y_{h(i)}} \right) \right\} \right]$$

$$MSE(\bar{y}_{strMM4}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h} \left[(S^2{}_{y_h} + R^2 \gamma_2{}^2 S^2{}_{x_h} - 2R\gamma_2 S_{x_h y_h}) - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_{h(i)}} - \gamma_2 D_{x_{h(i)}})^2 \right]$$

Where, $\lambda_1 = \dfrac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h(\bar{X}_h + C_{x_h})}$, $\lambda_2 = \dfrac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h\left(\bar{X}_h + \beta_{2h}(x)\right)}$, $\lambda_3 = \dfrac{\sum_{h=1}^{L} W_h \bar{X}_h \beta_{2h}(x)}{\sum_{h=1}^{L} W_h\left(\bar{X}_h \beta_{2h}(x) + C_{x_h}\right)}$,

$$\lambda_4 = \frac{\sum_{h=1}^{L} W_h \bar{X}_h C_{x_h}}{\sum_{h=1}^{L} W_h\left(\bar{X}_h C_{x_h} + \beta_{2h}(x)\right)}$$

In the present paper, a ratio estimator under StRSS is proposed which is more efficient than some of their counterpart ratio estimators.

3. Proposed Estimator

Motivated from Mandowra and Mehta (2014) we proposed ratio type estimator under stratified ranked set sampling. We use the first and third quartile as an auxiliary variable in our proposed estimator. The proposed estimator is as follows;

$$\bar{y}_{strss} = \bar{y}_{(StRSS)}\left(\frac{\sum_{h=1}^{L} W_h(\bar{X}_h + q_n)}{\sum_{h=1}^{L} W_h(\bar{x}_{h(rss)} + q_n)}\right)$$

Where, n=1, 3. To derive the Bias and MSE of the proposed estimator, we process as follows,

$$\bar{y}_{(strss)} = \bar{Y}(1+e_0), \bar{x}_{(strss)} = \bar{Y}(1+e_1), E(e_0) = E(e_1) = 0$$

$$E(e_0^2) = \sum_{h=1}^{L} \frac{W_h^{\ 2}}{n_h}\left[\frac{S^2_{x_h}}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_{h(i)}}^{\ 2}\right]$$

$$E(e_0 e_1) = \sum_{h=1}^{L} \frac{W_h^{\ 2}}{n_h}\left[\frac{S_{x_h y_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_{h(i)} y_{h(i)}}\right]$$

$$E(e_1^2) = \sum_{h=1}^{L} \frac{W_h^{\ 2}}{n_h}\left[\frac{S^2_{y_h}}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{y_{h(i)}}^{\ 2}\right]$$

The proposed estimator can be written as under,

$$\frac{1}{1+\lambda_n e_1} = \frac{\sum_{h=1}^{L} W_h(\bar{X}_h + q_n)}{\sum_{h=1}^{L} W_h(\bar{x}_{h(rss)} + q_n)}$$

$$\lambda_n e_1 = \frac{\sum_{h=1}^{L} W_h(\bar{x}_{h(rss)} + q_n) - \sum_{h=1}^{L} W_h(\bar{X}_h - q_n)}{\sum_{h=1}^{L} W_h(\bar{X}_h + q_n)}$$

$$\lambda_n = \frac{\sum_{h=1}^{L} W_h(\bar{x}_{h(rss)} - \bar{X}_h)}{\sum_{h=1}^{L} W_h(\bar{X}_h + q_n)} \quad \frac{\bar{X}_h}{\bar{x}_{h(rss)} - \bar{X}_h}$$

$$\lambda_n = \frac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h(\bar{X}_h + q_n)}$$

The bias of our proposed estimator is derived as below

$$Bias\ (\bar{y}_{strS}) = E(\bar{y}_{strS}) - \bar{Y}$$

Here,

$$\bar{y}_{strS} = \bar{Y}(1+e_0)(1+\lambda_n e_1)^{-1}$$

Neglecting the terms of higher order,

$$\bar{y}_{strS} = \bar{Y}(1+e_0)(1-\lambda_n e_1 + \lambda_n{}^2 e_1{}^2)$$

$$\bar{y}_{strS} - \bar{Y} = \bar{Y}e_0 - \bar{Y}\lambda_n e_1 - \bar{Y}\lambda_n e_0 e_1 + \bar{Y}\lambda_n{}^2 e_1^2$$

Taking expectation on both sides,

$$E(\bar{y}_{strS} - \bar{Y}) = \bar{Y}E(e_0) - \bar{Y}\lambda_n E(e_1) - \bar{Y}\lambda_n E(e_0 e_1) + \bar{Y}\lambda_n{}^2 E(e_1{}^2)$$

$$Bias\ (\bar{y}_{strS}) = -\bar{Y}\lambda_n \sum_{h=1}^{L} \frac{W_h{}^2}{n_h}\left[\frac{S_{x_h y_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_{h(i)} y_{h(i)}}\right] + \bar{Y}\lambda_n{}^2 \sum_{h=1}^{L}\frac{W_h{}^2}{n_h}\left[\frac{S^2{}_{x_h}}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_{h(i)}}{}^2\right]$$

$$Bias\ (\bar{y}_{strS})\bar{Y}\left[\sum_{h=1}^{L}\frac{W_h{}^2}{n_h}\left\{\frac{\lambda^2{}_n S^2{}_{x_h}}{\bar{X}^2} - \frac{\lambda_n S^2{}_{x_h y_h}}{\bar{X}\bar{Y}}\right\} - \sum_{h=1}^{L}\frac{W_h{}^2}{n_h}\left\{\frac{m}{n_h}\left(\lambda^2{}_n \sum_{i=1}^{r_h} D^2{}_{x_{h(i)}} - \lambda_n \sum_{i=1}^{r_h} D_{x_{h(i)} y_{h(i)}}\right)\right\}\right]$$

The MSE of our proposed estimator is derived as below

$$E(\bar{y}_{strS} - \bar{Y})^2 = E(\bar{Y}e_0 - \bar{Y}\lambda_n e_1)^2$$

$$MSE(\bar{y}_{strS}) = E(\bar{y}_{strS} - \bar{Y})^2 = \bar{Y}^2\{E(e_0{}^2) + \lambda_n{}^2 E(e_1{}^2) - 2\lambda_n E(e_0 e_1)\}$$

$$MSE(\bar{y}_{strS}) = \bar{Y}^2\left[\sum_{h=1}^{L}\frac{W_h{}^2}{n_h}\left(\frac{S^2{}_{y_h}}{\bar{Y}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D^2{}_{y_{h(i)}}\right) + \lambda^2{}_n \sum_{h=1}^{L}\frac{W_h{}^2}{n_h}\left(\frac{S^2{}_{x_h}}{\bar{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D^2{}_{x_{h(i)}}\right) - 2\lambda_n \sum_{h=1}^{L}\frac{W_h{}^2}{n_h}\left(\frac{S_{x_h y_h}}{\bar{X}\bar{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_{h(i)} y_{h(i)}}\right)\right]$$

$$MSE(\bar{y}_{strS}) = \sum_{h=1}^{L}\frac{W_h{}^2}{n_h}\left[(S^2{}_{y_h} + R^2\lambda^2{}_n S^2{}_{x_h} - 2R\lambda_n S_{x_h y_h}) - \bar{Y}^2 \frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_{h(i)}} - \lambda_n D_{x_{h(i)}})^2\right]$$

Efficiency Comparison with Kadilar and Cingi (2003),

$$\text{MSE}\ \bar{\text{y}}_{str\ \text{S}} \le \text{MSE}\ \bar{\text{y}}_{str\ \text{SD}}$$

$$\sum_{h=1}^{L}\frac{W_h{}^2}{n_h}\left[\left(S_{y_h}{}^2 + R^2\lambda_n{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h}\right) - \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_n D_{x_h(i)})^2\right] \le \sum_{h=1}^{L}\frac{W_h{}^2}{n_h}\left(S_{y_h}{}^2 + R^2\lambda_1{}^2 S_{x_h}{}^2 - 2R\lambda_1 S_{x_h y_h}\right)$$

$$R^2\lambda_n{}^2 S_{x_h}{}^2 - R^2\lambda_1{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h} + 2R\lambda_1 S_{x_h y_h} - \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \le 0$$

$$R^2 S_{x_h}{}^2(\lambda_n + \lambda_1)(\lambda_n - \lambda_1) - 2RS_{x_h y_h}(\lambda_n - \lambda_1) - \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \le 0$$

$$R^2 S_{x_h}{}^2 (\lambda_n + \lambda_1)(\lambda_n - \lambda_1) - 2RS_{x_h y_h}(\lambda_n - \lambda_1) - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \leq 0$$

$$(\lambda_n - \lambda_1)\left[R^2 S_{x_h}{}^2 (\lambda_n + \lambda_1) - 2RS_{x_h y_h}\right] - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \leq 0$$

$$A_1 \leq 0$$

Where,

$$A_1 = (\lambda_n - \lambda_1)\left[R^2 S_{x_h}{}^2 (\lambda_n + \lambda_1) - 2RS_{x_h y_h}\right] - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2$$

Efficiency Comparison with Second Estimator of Kadilar and Cingi (2003),

$$\text{MSE } \bar{y}_{str\ \text{S}} \leq \text{MSE } \bar{y}_{str\ \text{SK2}}$$

$$\sum_{h=1}^{L} \frac{W_h{}^2}{n_h}\left[\left(S_{y_h}{}^2 + R^2 \lambda_n{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h}\right) - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2\right] \leq \sum_{h=1}^{L} \frac{W_h{}^2}{n_h}\left(S_{y_h}{}^2 + R^2 \lambda_2{}^2 S_{x_h}{}^2 - 2R\lambda_2 S_{x_h y_h}\right)$$

$$R^2 \lambda_n{}^2 S_{x_h}{}^2 - R^2 \lambda_2{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h} + 2R\lambda_2 S_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \leq 0$$

$$R^2 S_{x_h}{}^2 (\lambda_n + \lambda_2)(\lambda_n - \lambda_2) - 2RS_{x_h y_h}(\lambda_n - \lambda_2) - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \leq 0$$

$$R^2 S_{x_h}{}^2 (\lambda_n + \lambda_2)(\lambda_n - \lambda_2) - 2RS_{x_h y_h}(\lambda_n - \lambda_2) - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \leq 0$$

$$(\lambda_n - \lambda_1)\left[R^2 S_{x_h}{}^2 (\lambda_n + \lambda_2) - 2RS_{x_h y_h}\right] - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \leq 0$$

$$A_2 \leq 0$$

where,

$$A_2 = (\lambda_n - \lambda_2)\left[R^2 S_{x_h}{}^2 (\lambda_n + \lambda_2) - 2RS_{x_h y_h}\right] - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2$$

Efficiency Comparison with Upadhyaya and Singh (1999),

$$\text{MSE } \bar{y}_{str\ \text{S}} \leq \text{MSE } \bar{y}_{str\ \text{US1}}$$

$$\sum_{h=1}^{L} \frac{W_h{}^2}{n_h}\left[\left(S_{y_h}{}^2 + R^2 \lambda_n{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h}\right) - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2\right] \leq \sum_{h=1}^{L} \frac{W_h{}^2}{n_h}\left(S_{y_h}{}^2 + R^2 \gamma_1{}^2 S_{x_h}{}^2 - 2R\gamma_1 S_{x_h y_h}\right)$$

$$R^2 \lambda_n{}^2 S_{x_h}{}^2 - R^2 \gamma_1{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h} + 2R\gamma_1 S_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \leq 0$$

$$R^2 S_{x_h}{}^2 (\lambda_n + \gamma_1)(\lambda_n - \gamma_1) - 2RS_{x_h y_h}(\lambda_n - \gamma_1) - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \leq 0$$

$$R^2 S_{x_h}{}^2 (\lambda_n + \gamma_1)(\lambda_n - \gamma_1) - 2RS_{x_h y_h}(\lambda_n - \gamma_1) - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \leq 0$$

$$(\lambda_n - \gamma_1)\left[R^2 S_{x_h}{}^2 (\lambda_n + \gamma_1) - 2RS_{x_h y_h}\right] - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \leq 0$$

$$A_3 \leq 0,$$

where,

$$A_3 = (\lambda_n - \gamma_1)\left[R^2 S_{x_h}{}^2(\lambda_n + \gamma_1) - 2RS_{x_h y_h}\right] - \bar{Y}^2 \frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_n D_{x_h(i)})^2$$

Efficiency Comparison with second estimator by Upadhyaya and Singh (1999),

$$\text{MSE } \bar{y}_{str\ \text{S}} \leq \text{MSE } \bar{y}_{str\ \text{US2}}$$

$$\sum_{h=1}^{L}\frac{W_h{}^2}{n_h}\left[\left(S_{y_h}{}^2 + R^2\lambda_n{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h}\right) - \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_n D_{x_h(i)})^2\right] \leq \sum_{h=1}^{L}\frac{W_h{}^2}{n_h}\left(S_{y_h}{}^2 + R^2\gamma_2{}^2 S_{x_h}{}^2 - 2R\gamma_2 S_{x_h y_h}\right)$$

$$R^2\lambda_n{}^2 S_{x_h}{}^2 - R^2\gamma_2{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h} + 2R\gamma_2 S_{x_h y_h} - \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \leq 0$$

$$R^2 S_{x_h}{}^2(\lambda_n + \gamma_2)(\lambda_n - \gamma_2) - 2RS_{x_h y_h}(\lambda_n - \gamma_2) - \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \leq 0$$

$$R^2 S_{x_h}{}^2(\lambda_n + \gamma_2)(\lambda_n - \gamma_2) - 2RS_{x_h y_h}(\lambda_n - \gamma_2) - \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \leq 0$$

$$(\lambda_n - \gamma_2)\left[R^2 S_{x_h}{}^2(\lambda_n + \gamma_2) - 2RS_{x_h y_h}\right] - \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \leq 0$$

$$A_4 \leq 0$$

where,

$$A_4 = (\lambda_n - \gamma_2)\left[R^2 S_{x_h}{}^2(\lambda_n + \gamma_2) - 2RS_{x_h y_h}\right] - \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_n D_{x_h(i)})^2$$

Efficiency Comparison of $\text{MSE } \bar{y}_{strss}$ and $\text{MSE } \bar{y}_{str\ \text{MM1}}$

$$\text{MSE } \bar{y}_{str\ \text{S}} \leq \text{MSE } \bar{y}_{str\ \text{MM1}}$$

$$\sum_{h=1}^{L}\frac{W_h{}^2}{n_h}\left[\left(S_{y_h}{}^2 + R^2\lambda_n{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h}\right) - \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_n D_{x_h(i)})^2\right] \leq \sum_{h=1}^{L}\frac{W_h{}^2}{n_h}\left[\left(S_{y_h}{}^2 + R^2\lambda_1{}^2 S_{x_h}{}^2 - 2R\lambda_1 S_{x_h y_h}\right) - \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_1 D_{x_h(i)})^2\right]$$

$$R^2\lambda_n{}^2 S_{x_h}{}^2 - R^2\lambda_1{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h} + 2R\lambda_1 S_{x_h y_h} \leq \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 - \bar{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_1 D_{x_h(i)})^2$$

$$R^2 S_{x_h}{}^2(\lambda_n + \lambda_1)(\lambda_n - \lambda_1) - 2RS_{x_h y_h}(\lambda_n - \lambda_1) \leq \bar{Y}^2\frac{m}{n_h}\left[D_{y_h}{}^2 + \lambda_n{}^2 D_{x_h}{}^2 - 2D_{y_h}\lambda_n D_{x_h} - D_{y_h}{}^2 - \lambda_1{}^2 D_{x_h}{}^2 + 2D_{y_h}\lambda_1 D_{x_h}\right]$$

$$R^2 S_{x_h}{}^2(\lambda_n + \lambda_1)(\lambda_n - \lambda_1) - 2RS_{x_h y_h}(\lambda_n - \lambda_1) \leq \bar{Y}^2\frac{m}{n_h}\left[D_{x_h}{}^2(\lambda_n + \lambda_1)(\lambda_n - \lambda_1) - 2D_{y_h}D_{x_h}(\lambda_n - \lambda_1)\right]$$

$$(\lambda_{P1} - \lambda_1)\left[R^2 S_{x_h}{}^2(\lambda_{P1} + \lambda_1) - 2RS_{x_h y_h}\right] \leq \bar{Y}^2\frac{m}{n_h}\left[D_{x_h}{}^2(\lambda_n + \lambda_1) - 2D_{y_h}D_{x_h}\right](\lambda_n - \lambda_1)$$

$$R^2 S_{x_h}{}^2(\lambda_n + \lambda_1) - 2RS_{x_h y_h} - \bar{Y}^2\frac{m}{n_h}D_{x_h}{}^2(\lambda_n + \lambda_1) \leq 2(RS_{x_h y_h} - \bar{Y}^2\frac{m}{n_h}D_{y_h}D_{x_h})$$

$$(\lambda_n + \lambda_1)(R^2 S_{x_h}{}^2 - \bar{Y}^2\frac{m}{n_h}D_{x_h}{}^2) \leq 2(RS_{x_h y_h} - \bar{Y}^2\frac{m}{n_h}D_{y_h}D_{x_h})$$

$$\frac{(\lambda_n + \lambda_1)(R^2 S_{x_h}{}^2 - \bar{Y}^2\frac{m}{n_h}D_{x_h}{}^2)}{2(RS_{x_h y_h} - \bar{Y}^2\frac{m}{n_h}D_{y_h}D_{x_h})} \leq 1$$

$$A_5 \leq 1,$$

$$where,\ A_5 = \frac{(\lambda_n + \lambda_1)(R^2 S_{x_h}{}^2 - \bar{Y}^2 \frac{m}{n_h} D_{x_h}{}^2)}{2(RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{y_h} D_{x_h})}$$

Efficiency Comparison of $\text{MSE}\ \bar{y}_{strss}$ and $\text{MSE}\ \bar{y}_{str\ \text{MM2}}$

$$\text{MSE}\ \bar{y}_{str\ \text{S}} \le \text{MSE}\ \bar{y}_{str\ \text{MM2}}$$

$$\sum_{h=1}^{L} \frac{W_h{}^2}{n_h} \left[\left(S_{y_h}{}^2 + R^2 \lambda_n{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h} \right) - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \right] \le \sum_{h=1}^{L} \frac{W_h{}^2}{n_h} \left[\left(S_{y_h}{}^2 + R^2 \lambda_2{}^2 S_{x_h}{}^2 - 2R\lambda_2 S_{x_h y_h} \right) - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_2 D_{x_h(i)})^2 \right]$$

$$R^2 \lambda_n{}^2 S_{x_h}{}^2 - R^2 \lambda_2{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h} + 2R\lambda_2 S_{x_h y_h} \le \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_2 D_{x_h(i)})^2$$

$$R^2 S_{x_h}{}^2 (\lambda_n + \lambda_2)(\lambda_n - \lambda_2) - 2RS_{x_h y_h} (\lambda_n - \lambda_2) \le \bar{Y}^2 \frac{m}{n_h} \left[D_{y_h}{}^2 + \lambda_n{}^2 D_{x_h}{}^2 - 2D_{y_h} \lambda_n D_{x_h} - D_{y_h}{}^2 - \lambda_2{}^2 D_{x_h}{}^2 + 2D_{y_h} \lambda_2 D_{x_h} \right]$$

$$R^2 S_{x_h}{}^2 (\lambda_n + \lambda_2)(\lambda_n - \lambda_2) - 2RS_{x_h y_h} (\lambda_n - \lambda_2) \le \bar{Y}^2 \frac{m}{n_h} \left[D_{x_h}{}^2 (\lambda_n + \lambda_2)(\lambda_n - \lambda_2) - 2D_{y_h} D_{x_h} (\lambda_n - \lambda_2) \right]$$

$$(\lambda_n - \lambda_2) \left[R^2 S_{x_h}{}^2 (\lambda_n + \lambda_2) - 2RS_{x_h y_h} \right] \le \bar{Y}^2 \frac{m}{n_h} \left[D_{x_h}{}^2 (\lambda_n + \lambda_2) - 2D_{y_h} D_{x_h} \right] (\lambda_n - \lambda_2)$$

$$R^2 S_{x_h}{}^2 (\lambda_n + \lambda_2) - 2RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{x_h}{}^2 (\lambda_n + \lambda_2) \le 2(RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{y_h} D_{x_h})$$

$$(\lambda_n + \lambda_2)(R^2 S_{x_h}{}^2 - \bar{Y}^2 \frac{m}{n_h} D_{x_h}{}^2) \le 2(RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{y_h} D_{x_h})$$

$$\frac{(\lambda_n + \lambda_2)(R^2 S_{x_h}{}^2 - \bar{Y}^2 \frac{m}{n_h} D_{x_h}{}^2)}{2(RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{y_h} D_{x_h})} \le 1$$

$$A_6 \le 1$$

Where,

$$A_6 = \frac{(\lambda_n + \lambda_2)(R^2 S_{x_h}{}^2 - \bar{Y}^2 \frac{m}{n_h} D_{x_h}{}^2)}{2(RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{y_h} D_{x_h})}$$

Efficiency Comparison with of $\text{MSE}\ \bar{y}_{strss}$ and $\text{MSE}\ \bar{y}_{str\ \text{MM3}}$

$$\text{MSE}\ \bar{y}_{str\ \text{S}} \le \text{MSE}\ \bar{y}_{str\ \text{MM3}}$$

$$\sum_{h=1}^{L} \frac{W_h{}^2}{n_h} \left[\left(S_{y_h}{}^2 + R^2 \lambda_n{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h} \right) - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 \right] \le \sum_{h=1}^{L} \frac{W_h{}^2}{n_h} \left[\left(S_{y_h}{}^2 + R^2 \gamma_1{}^2 S_{x_h}{}^2 - 2R\gamma_1 S_{x_h y_h} \right) - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \gamma_1 D_{x_h(i)})^2 \right]$$

$$R^2 \lambda_n{}^2 S_{x_h}{}^2 - R^2 \gamma_1{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h} + 2R\gamma_1 S_{x_h y_h} \le \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 - \bar{Y}^2 \frac{m}{n_h} \sum_{i=1}^{r_h} (D_{y_h(i)} - \gamma_1 D_{x_h(i)})^2$$

$$R^2 S_{x_h}{}^2 (\lambda_n + \gamma_1)(\lambda_n - \gamma_1) - 2RS_{x_h y_h} (\lambda_n - \gamma_1) \le \bar{Y}^2 \frac{m}{n_h} \left[D_{y_h}{}^2 + \lambda_n{}^2 D_{x_h}{}^2 - 2D_{y_h} \lambda_n D_{x_h} - D_{y_h}{}^2 - \gamma_1{}^2 D_{x_h}{}^2 + 2D_{y_h} \gamma_1 D_{x_h} \right]$$

$$R^2 S_{x_h}{}^2 (\lambda_n + \gamma_1)(\lambda_n - \gamma_1) - 2RS_{x_h y_h} (\lambda_n - \gamma_1) \le \bar{Y}^2 \frac{m}{n_h} \left[D_{x_h}{}^2 (\lambda_n + \gamma_1)(\lambda_n - \gamma_1) - 2D_{y_h} D_{x_h} (\lambda_n - \gamma_1) \right]$$

$$(\lambda_n - \gamma_1) \left[R^2 S_{x_h}{}^2 (\lambda_n + \gamma_1) - 2RS_{x_h y_h} \right] \le \bar{Y}^2 \frac{m}{n_h} \left[D_{x_h}{}^2 (\lambda_n + \gamma_1) - 2D_{y_h} D_{x_h} \right] (\lambda_n - \gamma_1)$$

$$R^2 S_{x_h}{}^2(\lambda_n+\gamma_1) - 2RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{x_h}{}^2(\lambda_n+\gamma_1) \le 2(RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{y_h} D_{x_h})$$

$$(\lambda_n+\gamma_1)(R^2 S_{x_h}{}^2 - \bar{Y}^2 \frac{m}{n_h} D_{x_h}{}^2) \le 2(RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{y_h} D_{x_h})$$

$$\frac{(\lambda_n+\gamma_1)(R^2 S_{x_h}{}^2 - \bar{Y}^2 \frac{m}{n_h} D_{x_h}{}^2)}{2(RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{y_h} D_{x_h})} \le 1$$

$$A_7 \le 1$$

Where,

$$A_7 = \frac{(\lambda_n+\gamma_1)(R^2 S_{x_h}{}^2 - \bar{Y}^2 \frac{m}{n_h} D_{x_h}{}^2)}{2(RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{y_h} D_{x_h})}$$

Efficiency Comparison of $\text{MSE}\ \bar{y}_{strss}$ and $\text{MSE}\ \bar{y}_{str\ \text{MM4}}$,

$$\text{MSE}\ \bar{y}_{str\ \text{S}} \le \text{MSE}\ \bar{y}_{str\ \text{MM4}}$$

$$\sum_{h=1}^{L} \frac{W_h{}^2}{n_h}\left[\left(S_{y_h}{}^2 + R^2\lambda_n{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h}\right) - \bar{Y}^2 \frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_n D_{x_h(i)})^2\right] \le \sum_{h=1}^{L} \frac{W_h{}^2}{n_h}\left[\left(S_{y_h}{}^2 + R^2\gamma_2{}^2 S_{x_h}{}^2 - 2R\gamma_2 S_{x_h y_h}\right) - \bar{Y}^2 \frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \gamma_2 D_{x_h(i)})^2\right]$$

$$R^2\lambda_n{}^2 S_{x_h}{}^2 - R^2\gamma_2{}^2 S_{x_h}{}^2 - 2R\lambda_n S_{x_h y_h} + 2R\gamma_2 S_{x_h y_h} \le \bar{Y}^2 \frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \lambda_n D_{x_h(i)})^2 - \bar{Y}^2 \frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h(i)} - \gamma_2 D_{x_h(i)})^2$$

$$R^2 S_{x_h}{}^2(\lambda_n+\gamma_2)(\lambda_n-\gamma_2) - 2RS_{x_h y_h}(\lambda_n-\gamma_2) \le \bar{Y}^2 \frac{m}{n_h}\left[D_{y_h}{}^2 + \lambda_n{}^2 D_{x_h}{}^2 - 2D_{y_h}\lambda_n D_{x_h} - D_{y_h}{}^2 - \gamma_2{}^2 D_{x_h}{}^2 + 2D_{y_h}\gamma_2 D_{x_h}\right]$$

$$R^2 S_{x_h}{}^2(\lambda_n+\gamma_2)(\lambda_n-\gamma_2) - 2RS_{x_h y_h}(\lambda_n-\gamma_2) \le \bar{Y}^2 \frac{m}{n_h}\left[D_{x_h}{}^2(\lambda_n+\gamma_2)(\lambda_n-\gamma_2) - 2D_{y_h} D_{x_h}(\lambda_n-\gamma_2)\right]$$

$$(\lambda_n-\gamma_2)\left[R^2 S_{x_h}{}^2(\lambda_n+\gamma_2) - 2RS_{x_h y_h}\right] \le \bar{Y}^2 \frac{m}{n_h}\left[D_{x_h}{}^2(\lambda_n+\gamma_2) - 2D_{y_h} D_{x_h}\right](\lambda_n-\gamma_2)$$

$$R^2 S_{x_h}{}^2(\lambda_n+\gamma_2) - 2RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{x_h}{}^2(\lambda_n+\gamma_2) \le 2(RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{y_h} D_{x_h})$$

$$(\lambda_n+\gamma_2)(R^2 S_{x_h}{}^2 - \bar{Y}^2 \frac{m}{n_h} D_{x_h}{}^2) \le 2(RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{y_h} D_{x_h})$$

$$\frac{(\lambda_n+\gamma_2)(R^2 S_{x_h}{}^2 - \bar{Y}^2 \frac{m}{n_h} D_{x_h}{}^2)}{2(RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{y_h} D_{x_h})} \le 1$$

$$A_8 \le 1$$

Where,

$$A_8 = \frac{(\lambda_n+\gamma_2)(R^2 S_{x_h}{}^2 - \bar{Y}^2 \frac{m}{n_h} D_{x_h}{}^2)}{2(RS_{x_h y_h} - \bar{Y}^2 \frac{m}{n_h} D_{y_h} D_{x_h})}$$

4. Conclusion

We proposed new ratio-type estimators for StRSS and obtained their bias and MSE. The MSE of our proposed estimators is compared to Kadilar and Cingi (2003), Upadhyaya and Singh (1999), Mandowara and Mehta (2014). We found that the proposed estimator has smaller MSE than the corresponding estimators under some conditions. Thus, it is concluded that the proposed ratio type estimator of the population mean using stratified ranked set sampling is more efficient than some of the existing estimators.

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