Analysis of a Reliability Model of an Off-Grid Solar System and a Generator Under Conditional Repair

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Abstract: In this paper, a reliability model of an off-grid solar system (subsystem-A) and a generator (subsystem-B) is developed to analyze the numerous reliability measures such as mean time to system failure, availability, busy period of the technician and profit of the system model. Subsystem-A is a primary source of electricity and is initially operative whereas subsystem-B is a backup unit in cold standby mode. A technician is hired to perform all the repair and associated activities who attends subsystem-A at its failure and starts its repair instantly, whereas subsystem-B undergoes inspection to identify the requirement of minor/major repair. Failure and repair rates are distributed exponentially while general distribution is taken for activation rate of subsystem-A and operation rate for both the subsystems. Semi-Markov process and regenerative point techniques are applied to find the reliability measures. For arbitrary values, MTSF and profit of the system model are analyzed, and their graphs are drawn to show the impact of conditional repair on the system model. From the outputs we observe that the conditional repair policy plays key role while optimizing the MTSF and profit of the system model.

Keywords: Reliability, Semi-Markov, Minor Repair, Major Repair, Activation, Inspection.

1. Introduction

The reliability of a system has played a vital role from the ages. We are using various types of systems from generations which affect our pocket to life. A more reliable system increases the size of our pockets and improves our living in various ways. Research and development perform a key role in improving the reliability of a system. Researchers including Osaki and Asakura (1970) used the concept of preventive maintenance to study a redundant system of two units. Srinivasan and Gopalan (1973) analysed a warm standby two-unit system using single repair facility. Nakagawa (1977) identified the optimum preventive maintenance policies of repairable systems. Gopalan and Marathe (1978) discussed the availability of a system of 1-server and 2-dissimilar units using a slow switch. Murari et al. (1985) conducted cost analysis of a warm standby system with two units, single regular repairman and waiting time. Singh et al. (1991) conducted cost-benefit analysis of a 2-unit standby system with the concept of priority and repair waiting time. Dhillon (1992) carried out the analysis of reliability and availability of a standby system with common cause failures. Mokaddis et al. (1997) studied a two-unit warm standby system with degradation. Kadyan et al. (2004) analysed non-identical unit reliability systems stochastically using the concept of priority and different modes of failure. Wang and Zhang (2007) used geometric process repair for a two-component series system to discuss the optimum replacement policy. Malik et al. (2008) stochastically analysed an operating system with two inspection policies and degradation. Yusuf and Koki (2013) evaluated reliability and availability characteristics of an active parallel units repairable system. Upma and Malik (2016) analysed the cost-benefit of a non-identical system with preventive repair and replacement. Kumar et al. (2019) discussed the profit of a non-identical unit warm standby system with single sever performance in different environmental conditions. Jain et al. (2020) measured the reliability of a two-unit standby system with delayed service. Kadyan et al. (2020) discussed a repairable system of three non-identical units with operational priority and simultaneous operation of cold standby units. Ghosh et al. (2022) developed and analysed a triple unit system model with operation priority. Kumar et al. (2022) analysed the cost effectiveness of a complex system under diverse repair policies in normal weather condition. Ram et al. (2022) analysed a stochastic model of a rework system. Ghosh et al. (2023) analysed the performance of a non-identical unit systems with inspection and operational priority. Recently, Kumar et at. (2024) performed the reliability assessment and optimization of complex systems.

Considering the importance of non-identical standby systems, here we developed a reliability model of two non-identical subsystems keeping off-grid solar system (subsystem-A) as primary and generator (subsystem-B) as secondary subsystem under the following assumptions:

- Initially, subsystem-A is operative whereas subsystem-B is cold-standby.
- Subsystem-A has three states – normally operative, under repair and under activation.
- Subsystem-B has four states – cold standby, normally operative, under inspection and under minor/major repair.
- System remains operative if anyone of the two subsystems is operative.
- A technician is always available with the system to deal with all kinds of faults.
- Precedence to repair and activate subsystem-A is given over subsystem-B.
- Subsystem-A undergoes repair immediately at failure and goes for activation if required whereas subsystem-B undergoes inspection at failure to identify the requirement of minor/major repair.
- Repair and Failure times of both subsystems are independent.
- Failure times of both subsystems, repair time of subsystem-A and inspection time of subsystem-B are exponentially distributed while general distribution is taken for activation time of subsystem-A and repair times of both the subsystems.
- MTSF and profit of the reliability model are analysed for various parameters and graphs are drawn to predict their behaviour.

Notations

A_o /A_r /A_a	:	Subsystem-A is normally operative/ under repair/ under activation.
B_o/B_{cs}	:	Subsystem-B is normally operative/ in cold standby mode.
B_{wi} / B_{wr1} / B_{Wr2}	:	Subsystem-B is failed and waiting for inspection/ minor repair/ major repair.
B_{fui} / B_{r1} / B_{r2}	:	Subsystem-B is failed and under under inspection/ minor repair/ major repair.
α_1/ λ_1/ λ_2	:	Consistent rate of failure of subsystem-A/ subsystem-B/ subsystem-A during activation.
θ	:	Subsystem-B is under inspection at constant rate.
a/ b	:	Probability of minor/ major repair of subsystem-B.
β_1	:	Constant repair rate of subsystem-A.
$G_1(\cdot)$/$G_2(\cdot)$	:	cdf of minor/ major repair time of subsystem-B.
$H_1(\cdot)$/$H_2(\cdot)$	:	cdf of activation/ repair time of subsystem-A.
$q_{i\,j}$	:	Transition rate of the system from state $(i - j)$ on or before time 't'.
ψ_i	:	Mean sojourn time in the i^{th} state, known as the probable stay time of the system in i^{th} state before transiting to another state. If T_i is the sojourn time in i^{th} state, then mean sojourn time is $\psi_i = \int P(T_i > t)\,dt$.
z_i	:	Probability that the system sojourns in state S_i up to time t.
m_{ij}	:	Mean sojourn time of the system in state S_i when the system, is to transit to regenerative state S_j i.e., $m_{ij} = \int t\, q_{ij}(t)dt$.
*/**	:	Symbol of Laplace Stieltjes Transformation (LST)/ Laplace Transformation (LT).
©/ ′(desh)	:	Symbols for Laplace convolution/ derivative of the function.
K_0	:	Revenue per unit up-time when system is operative.
K_1	:	Repairing cost per unit of time for subsystem-A.
K_2	:	Cost per unit of time when subsystem-A is under activation.
K_3	:	Cost per unit of time when subsystem-B is under inspection.
K_4	:	Repairing cost per unit of time for subsystem-B.
K_5	:	Fixed expanses per unit time paid to the server.

State transition diagram with transition rates are represented in figure -1 below:

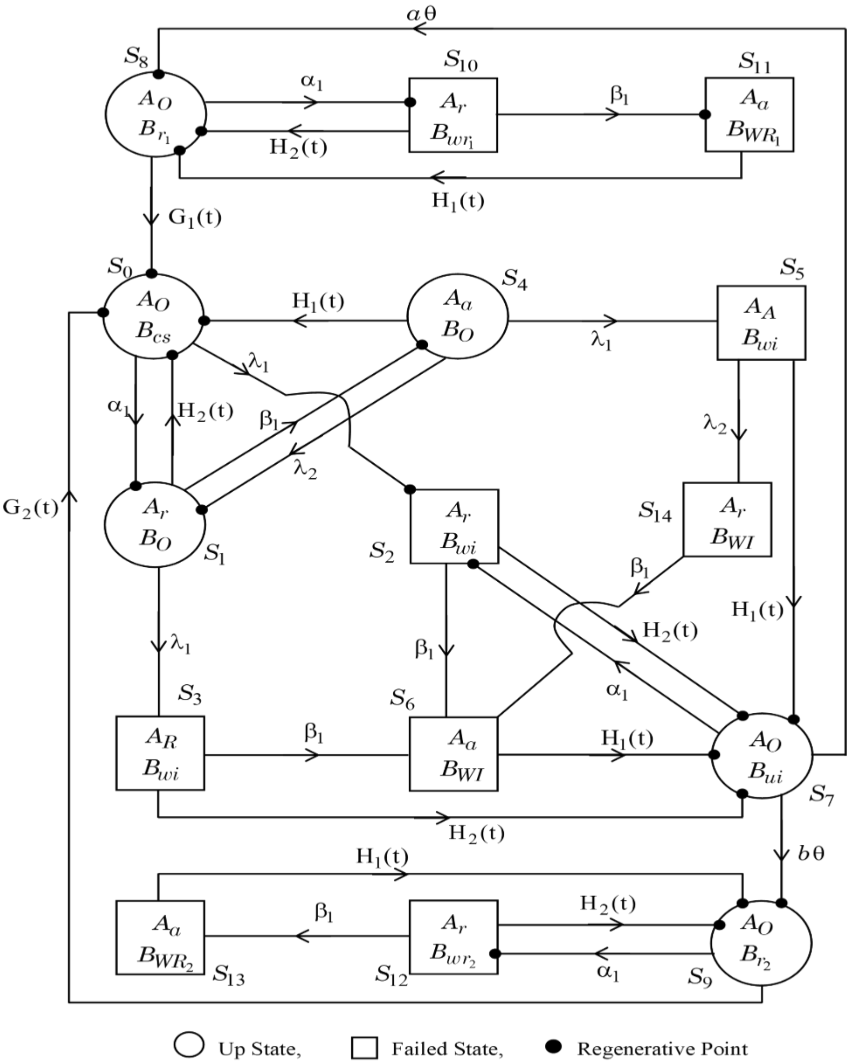

Figure 1: State Transition Diagram

2. Transition Probabilities and Mean Sojourn Times

The probabilities of steady-state transition are calculated by

$$p_{ij} = \lim_{t\to\infty} Q_{ij}(t); \quad p_{ij}^{(k)}(t) = \lim_{t\to\infty} Q_{ij}^{(k)}(t)$$

Therefore,

$p_{0,1} = \frac{\alpha_1}{\alpha_1+\lambda_1}$; $p_{0,2} = \frac{\lambda_1}{\alpha_1+\lambda_1}$; $p_{1,0} = \widetilde{H_2}(\lambda_1+\beta_1)$; $p_{1,4} = (1-p_{1,0})\frac{\beta_1}{\beta_1+\lambda_1}$; $p_{2,6} = [1-\widetilde{H_2}(\beta_1)] = p_{10,11} = p_{12,13}$; $p_{2,7} = \widetilde{H_2}(\beta_1) = p_{10,8} = p_{12,9}$; $p_{4,0} = \widetilde{H_1}(\lambda_1+\lambda_2)$; $p_{4,1} = (1-p_{4,0})\frac{\lambda_1}{\lambda_1+\lambda_2}$; $p_{6,7} = 1 = p_{11,8} = p_{13,9} = p_{14,6}$; $p_{7,2} = \frac{\alpha_1}{\alpha_1+\theta}$; $p_{7,8} = \frac{a\theta}{\alpha_1+\theta}$; $p_{7,9} = \frac{b\theta}{\alpha_1+\theta}$; $p_{8,0} = \widetilde{G_1}(\alpha_1)$; $p_{8,10} = (1-p_{8,0})$; $p_{9,0} = \widetilde{G_2}(\alpha_1)$; $p_{9,12} = (1-p_{9,0})$; $p_{1,6}^{(3)} = (p_{2,6}-p_{1,4})$; $p_{1,7}^{(3)} = (p_{2,7}-p_{1,0})$; $p_{4,7}^{(5)} = [\widetilde{H_1}(\lambda_2) - \widetilde{H_2}(\lambda_1+\lambda_2)]$; $p_{4,14}^{(5)} = [1-\widetilde{H_1}(\lambda_2)] - (1-p_{4,0})\frac{\lambda_2}{\lambda_1+\lambda_2}$

We observe that the following relations hold true:

$p_{0,1} + p_{0,2} = p_{1,0} + p_{1,4} + p_{1,6}^{(3)} + p_{1,7}^{(3)} = p_{2,6} + p_{2,7} = p_{4,0} + p_{4,1} + p_{4,7}^{(5)} + p_{4,14}^{(5)} = p_{6,7} = p_{7,2} + p_{7,8} + p_{7,9} = p_{80} + p_{8,10} = p_{9,0} + p_{9,12} = p_{10,8} + p_{10,11} = p_{11,8} = p_{12,9} + p_{12,.13} = p_{13,9} = p_{14,6} = p_{1,0} + p_{1,4} + p_{1,6}^{(3)} + p_{1,7}^{(3)} = p_{4,0} + p_{4,1} + p_{4,7}^{(5)} + p_{4,14}^{(5)} = 1$

2.1 Mean Sojourn Time (ψ_i) in State (S_i) are:
$\psi_0 = \frac{1}{\alpha_1+\lambda_1}$; $\psi_1 = \int e^{-(\lambda_1+\beta_1)u}\,\overline{H_2}(u)du$; $\psi_2 = \int e^{-\beta_1 u}\,\overline{H_2}(u)du = \psi_{10} = \psi_{12}$; $\psi_4 = \int e^{-(\lambda_1+\lambda_2)u}\,\overline{H_1}(u)du$; $\psi_6 = \int \overline{H_1}(u)du = \psi_{11} = \psi_{13}$; $\psi_7 = \frac{1}{\alpha_1+\theta}$; $\psi_8 = \int e^{-\alpha_1 u}\,\overline{G_1}(u)du$; $\psi_9 = \int e^{-\alpha_1 u}\,\overline{G_2}(u)du$; $\psi_{14} = \frac{1}{\beta_1}$

Now, mean sojourn time $\left(m_{ij}\right)$ of the system in state (S_i) before transiting to S_j (regenerative state) is given by $m_{ij} = \int t\,d\theta_{ij}(t) = \int t\,q_{ij}(t)dt$

So, we have $m_{01} = \frac{\alpha_1}{(\alpha_1+\lambda_1)^2}$; $m_{0,2} = \frac{\lambda_1}{(\alpha_1+\lambda_1)^2}$; $m_{1,0} = \int tdH_2(t)e^{-(\lambda_1+\beta_1)u}$; $m_{1,4} = \int t\beta_1 e^{-(\beta_1+\lambda_1)u}\overline{H_2}(u)\,du$; $m_{2,6} = \int t\beta_1 e^{-\beta_1 u}\overline{H_2}(u)\,du = m_{10,11} = m_{12,13}$; $m_{2,7} = \int tdH_2(u)e^{-\beta_1 u}\,du$; $m_{4,1} = \int t\lambda_2 e^{-(\lambda_1+\lambda_2)u}\overline{H_1}(u)\,du$; $m_{4,0} = \int tdH_1(u)e^{-(\lambda_1+\lambda_2)u}$; $m_{6,7} = \int tdH_1(u) = m_{11,8}$; $m_{7,2} = \frac{\alpha_1}{(\alpha_1+\theta)^2}$; $m_{7,8} = \frac{a\theta}{(\alpha_1+\theta)^2}$; $m_{7,9} = \frac{b\theta}{(\alpha_1+\theta)^2}$; $m_{8,0} = \int tdG_1(u)e^{-\alpha_1 u}$; $m_{8,10} = \int t\alpha_1 e^{-\alpha_1 u}\overline{G_1}(u)du$; $m_{9,0} = \int tdG_2(u)e^{-\alpha_1 u}$; $m_{9,12} = \int t\alpha_1 e^{-\alpha_1 u}\overline{G_2}(u)du$; $m_{10,8} = \int tdH_2(u)e^{-\beta_1 u} = m_{12,9}$; $m_{13,9} = \int tdH_1(u)du$; $m_{14,6} = \frac{1}{\beta^2}$

The following relations for m_{ij}'s are obtained:
$m_{01} + m_{02} = \psi_0$; $m_{10} + m_{14} + m_{16}^{(3)} + m_{17}^{(3)} = n_1$; $m_{26} + m_{27} = \psi_2 = m_{67}$; $m_{40} + m_{41} + m_{4,7}^{(5)} + m_{4,14}^{(5)} = n_2$; $m_{72} + m_{78} + m_{79} = \psi_7$; $m_{80} + m_{8.10} = \psi_8$; $m_{90} + m_{9,12} = \psi_9$; $m_{10,8} + m_{10,11} = \psi_{10}$; $m_{11,8} = \psi_{11}$; $m_{12,9} + m_{12,13} = \psi_{12}$; $m_{13,9} = \psi_{13}$; $m_{14,6} = \psi_{14}$

3. System Reliability and MTSF

To determine the reliability $R_i(t)$,when the system initially starts from $S_i \in E$, we assume the failed states $S_2, S_3, S_5, S_6, S_{10}, S_{11}, S_{12}, S_{13}, S_{14}$ of the system as absorbing. By simple probabilistic arguments, we see that $R_0(t)$ is the summation of the contingencies given below:

(i) The system remains operative in the state 'i', without transiting to state 'j' up to time 't'. The contingency probability is: $e^{-(\alpha_1+\lambda_1)t} = z_0(t)$ (say)

(ii) The contingency probability during $(u, u + du), u \leq t$; the system remains operative in state S_1 and remains operative for time $(t - u)$ is given by:
$\int_0^t q_{0,1}(u)\,du R_1(t-u) = q_{0,1}(t)©R_1(t)$

Thus, we have
$R_0(t) = z_0(t) + q_{0,1}(t)©R_1(t)$; $R_1(t) = z_1(t) + q_{1,0}(t)©R_0(t) + q_{1,4}(t)©R_4(t)$;
$R_4(t) = z_4(t) + q_{4,0}(t)©R_0(t) + q_{4,1}(t)©R_1(t)$ (1)
where, $z_0(t) = e^{-(\lambda_1+\alpha_1)t}$; $z_1(t) = e^{-(\lambda_1+\beta_1)t}\overline{H_2}(t)$; $z_4(t) = e^{-(\lambda_1+\lambda_2)t}\overline{H_1}(t)$

Taking LT of the of the above set of relations and solving for $R_0^*(s)$, we get,

$$R_0^*(s) = N_1 / D_1$$

Using the ILT, we determine the system's reliability when it initially starts from S_0. MTSF of the system is given by $\lim_{s\to 0} R_0^*(s)$. Therefore

$$\mathrm{MTSF} = \psi_0 + p_{0,1}\left(\psi_1 + \psi_4 + p_{1,4}\right) / \left(1 + p_{0,1}p_{1,0} - p_{1,4}p_{4,1} + p_{0,1}p_{1,0}p_{4,1}\right)$$

4. Steady-State Availability

Let $A_i(t)$ be the probability that the system is operational at time 't', when it starts initially from $S_i \in$ E. By taking LT of $A_0(t)$ we get $A_0^*(s)$. The steady-state availability of the model is determined by:

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s\,A_0^*(s) = \frac{N_2}{D_2'}$$

where, $N_1 = \psi_0 + p_{0,1}(\psi_1 + \psi_4 + p_{1,4})$ and
$D'_2 = \{(1 - p_{14}p_{41})(1 - p_{72})p_{80}p_{90}\}\psi_0 + \{(1 - p_{72})p_{01}p_{80}p_{90}\}n_1 + \{[(1 - p_{01}p_{10} - p_{14}p_{41} - p_{01}p_{14}p_{40})\,p_{72} + p_{02}(1 - p_{14}p_{41})]p_{80}p_{90}\}\psi_2 + \{(1 - p_{72})p_{01}p_{14}p_{80}p_{90}\}n_2 + \{[(1 - p_{01}p_{10} -$

$$p_{14}p_{41} - p_{01}p_{14}p_{40})p_{72} + p_{02}(1 - p_{14}p_{41})]p_{26}p_{80}p_{90}\}\psi_6 + \{(1 - p_{01}p_{10} - p_{14}p_{41} - p_{01}p_{14}p_{40})p_{80}p_{90}\}\psi_7 + \{(1 - p_{0,1}p_{10} - p_{14}p_{41} - p_{01}p_{14}p_{40})p_{78}p_{90}\}\psi_8 + \{(1 - p_{01}p_{10} - p_{14}p_{41} - p_{01}p_{14}p_{40})p_{79}p_{80}\}\psi_9 + \{(1 - p_{01}p_{10} - p_{14}p_{41} - p_{01}p_{14}p_{40})p_{78}p_{80}\, p_{90}\}\psi_{10} + \{(1 - p_{01}p_{10} - p_{14}p_{41} - p_{01}p_{14}p_{40})p_{78}p_{8,10}p_{10,11}p_{90}\}\psi_{11} + \{(1 - p_{01}p_{10} - p_{14}p_{41} - p_{01}p_{14}p_{40})p_{79}p_{80}p_{9,12}\}\psi_{12} + \{(1 - p_{01}p_{10} - p_{14}p_{41} - p_{01}p_{14}p_{40})p_{79}p_{80}p_{9,12}p_{12,13}\}\psi_{13} + \left\{p_{01}p_{14}p_{4,14}^{(5)}p_{90}\right\}\psi_{14}$$

The probable operation time of the model during (0, t) due to a subsystem is given by:

$$u_{up}(t) = \int_0^t A_0(u)du$$

So that, $u_{up}^*(s) = \int_0^t \frac{A_0^*(s)}{(s)}$

5. Busy Period Analysis

Let $B_i^A(t)$, $B_i^a(t)$, $B_i^i(t)$ and $B_i^B(t)$ are the probabilities that the technician is engaged in repairing the failed subsystem-A, activation of subsystem-A, inspection of failed subsystem-B and minor/major repair of failed subsystem-B respectively at ‘t’, initially when the system starts from $S_i \in E$. Using LT , we can obtain $B_i^{A*}(s)$, $B_i^{a*}(s)$, $B_i^{i*}(s)$ and $B_i^{B*}(s)$.

In a long duration, the probability that the technician will be engaged in the repair of the failed subsystem-A, activation of subsystem-A, inspection of failed subsystem-B and minor/major repair of failed subsystem-B respectively is given by,

$B_0^A = \frac{N_3}{D'_2}$ $B_0^a = \frac{N_4}{D'_2}$ $B_0^i = \frac{N_5}{D'_2}$ and $B_0^B = \frac{N_6}{D'_2}$

where,

$$N_3 = \left[(\psi_1 + p_{13}\psi_3)(1 - p_{72})\left\{\left(p_{16}^{(3)} + p_{17}^{(3)}\right)p_{01}p_{72} + \left(p_{4,7}^{(5)} + p_{4,14}^{(5)}\right)p_{01}p_{14}p_{72} + p_{02} - p_{02}p_{14}p_{72}\right\}\psi_2 + p_{01}p_{14}(1 - p_{72})p_{4,14}^{(5)}\psi_{14}\right](1 - p_{8,10})(1 - p_{9,12}) + \left[\left(p_{4,7}^{(5)} - p_{4,14}^{(5)}\right)p_{01}p_{14} + \left(p_{16}^{(3)} + p_{17}^{(3)}\right)p_{01} + (p_{02} - p_{01}p_{14}p_{41})\right]p_{78}\psi_{10}p_{8,10}(1 - p_{9,12})\left[\left(p_{4,7}^{(5)} + p_{4,14}^{(5)}\right)p_{01}p_{14} + \left(p_{16}^{(3)} + p_{17}^{(3)}\right)p_{01} + (p_{02} - p_{02}p_{14}p_{41})\right]p_{79}(1 - p_{8,10})\psi_{12}p_{9,12}$$

$$N_4 = \left[\left(p_{4,7}^{(5)} - p_{4,14}^{(5)}\right)p_{01}p_{14} + \left(p_{16}^{(3)} + p_{17}^{(3)}\right)p_{01} + (p_{02} - p_{02}p_{14}p_{41})\right]\{p_{78}p_{10,11}\psi_{11}p_{8,10}(1 - p_{9,12}) + p_{79}(1 - p_{8,10})p_{12,13}\psi_{13}p_{9,12}\}$$

$$N_5 = \left[\left\{p_{01}p_{14} + \left(p_{16}^{(3)} + p_{17}^{(3)}\right)p_{01} - (p_{02} - p_{02}p_{14}p_{41})\right\}\psi_7\right](1 - p_{8,10})(1 - p_{9,12})$$

$$N_6 = \left[\left(p_{4,7}^{(5)} + p_{4,14}^{(5)}\right)p_{01}p_{14} + \left(p_{16}^{(3)} + p_{17}^{(3)}\right)p_{01} + (p_{02} - p_{02}p_{14}p_{41})\right]\{p_{78}\psi_8(1 - p_{9,12}) + p_{79}(1 - p_{8,10})\psi_9\}$$

and D'_2 is already defined.

The expected engagement period of the technician in repairing of failed subsystem-A, activation of subsystem-A, inspection of failed subsystem-B, and minor/major repairing of subsystem-B respectively during (0, t) are separately given by

$\mu_b^{A*}(t) = \frac{B_0^{A*}(s)}{s}$; $\mu_b^{a*}(s) = \frac{B_0^{a*}(s)}{s}$; $\mu_b^{i*}(s) = \frac{B_0^{i*}(s)}{s}$ and $\mu_b^{B*}(s) = \frac{B_0^{B*}(s)}{s}$

6. Cost-Benefit Analysis

Consider the expected operation time of the system when it is up and expected busy period of the technician when he is engaged in inspection, repair and activation of the failed subsystems during (0, t), then the expected profit of the system is given by,

P(t) = Probable total revenue generated in (0, t) – probable total cost of repair in (0, t)

$= K_0\mu_{up}(t) - K_1\mu_b^A(t) - K_2\mu_b^a(t) - K_3\mu_b^i(t) - K_4\mu_b^a(t) - K_5$

The probable profit per unit time in the steady-state is given by:

$$P = K_0A_0 - K_1B_0^A - K_2B_0^a - K_3B_0^i - K_4B_0^B - K_5$$

where A_0, B_0^A, B_0^a, B_0^i and B_0^B have been already defined.

7. Particular Case

To study the system's behaviour, reliability and profit functions are obtained by taking all the repair and activation times as negative exponential, i.e.,

$H_1(t) = 1 - e^{-\alpha_2 t}$; $H_2(t) = 1 - e^{-\alpha_3 t}$; $G_1(t) = 1 - e^{-\beta_2 t}$; $G_2(t) = 1 - e^{-\beta_3 t}$

In steady-state transition probabilities and mean sojourn times, we have the changes as:

$p_{1,0} = \frac{\alpha_3}{\lambda_1+\beta_1+\alpha_3}$; $p_{1,4} = \frac{\beta_1}{\beta_1+\lambda_1+\alpha_3}$; $p_{2,6} = \frac{\beta_1}{\beta_1+\alpha_3} = p_{10,11} = p_{12,13}$; $p_{2,7} = \frac{\alpha_3}{\alpha_3+\beta_1} = p_{10,8} = p_{12,9}$; $p_{4,0} = \frac{\alpha_2}{\alpha_2+\lambda_1+\lambda_2}$; $p_{4,1} = \frac{\lambda_2}{\lambda_1+\lambda_2+\alpha_2}$; $p_{6,7} = 1 = p_{11,8} = p_{13,9} = p_{14,6}$; $p_{8,0} = \frac{\beta_2}{\alpha_1+\beta_2}$; $p_{8,10} = \frac{\alpha_1}{\alpha_1+\beta_2}$; $p_{9,0} = \frac{\beta_3}{\beta_3+\alpha_1}$; $p_{9,12} = \frac{\alpha_1}{\alpha_1+\beta_3}$; $p_{1,6}^{(3)} = p_{2,6} - p_{1,4}$; $p_{1,7}^{(3)} = p_{2,7} - p_{1,0}$; $p_{4,7}^{(5)} = \frac{\alpha_2}{\lambda_2+\alpha_2} - \frac{\alpha_2}{\lambda_1+\lambda_2+\alpha_2}$; $p_{4,14}^{(5)} = \frac{\lambda_2}{\lambda_2+\alpha_2} - \frac{\lambda_2}{\lambda_1+\lambda_2+\alpha_2}$

$\psi_1 = \frac{1}{\lambda_1+\beta_1+\alpha_3}$; $\psi_2 = \frac{1}{\beta_1+\alpha_3} = \psi_{10} = \psi_{12}$; $\psi_4 = \frac{1}{\lambda_1+\lambda_2+\alpha_2}$; $\psi_6 = \frac{1}{\alpha_2} = \psi_{11} = \psi_{13}$; $\psi_8 = \frac{1}{\alpha_1+\beta_2}$; $\psi_9 = \frac{1}{\alpha_1+\beta_3}$

$m_{1,0} = \frac{\alpha_3}{(\lambda_1+\beta_1+\alpha_3)^2}$; $m_{1,4} = \frac{\beta_1}{(\beta_1+\lambda_1+\alpha_3)^2}$; $m_{2,6} = \frac{\beta_1}{(\beta_1+\alpha_3)^2} = m_{10,11} = m_{12,13}$; $m_{2,7} = \frac{\alpha_3}{(\alpha_3+\beta_1)^2} = m_{10,8} = m_{12,9}$; $m_{4,1} = \frac{\lambda_2}{(\lambda_1+\lambda_2+\alpha_2)^2}$; $m_{4,0} = \frac{\alpha_2}{(\lambda_1+\lambda_2+\alpha_2)^2}$; $m_{6,7} = \frac{1}{\alpha_2} = m_{11,8} = m_{13,9}$; $m_{8,0} = \frac{\beta_2}{(\beta_2+\alpha_1)^2}$; $m_{8,10} = \frac{\alpha_1}{(\alpha_1+\beta_2)^2}$; $m_{9,0} = \frac{\beta_3}{(\beta_3+\alpha_1)^2}$; $m_{9,12} = \frac{\alpha_1}{(\alpha_1+\beta_3)^2}$; $m_{14,6} = \frac{1}{\beta_1}$; $m_{1,6}^{(3)} = m_{2,6} - m_{1,4}$; $m_{1,7}^{(3)} = m_{2,7} - m_{1,0}$; $m_{4,7}^{(5)} = \frac{\alpha_2}{(\lambda_1+\alpha_2)^2} - \frac{\alpha_2}{(\lambda_1+\lambda_2+\alpha_2)^2}$; $m_{4,14}^{(5)} = \frac{\lambda_2}{(\lambda_1+\lambda_2)^2} - \frac{\lambda_2}{(\lambda_1+\lambda_2+\alpha_2)^2}$

8. Discussion

To predict the behaviour of the considered reliability model, graphs for MTSF and cost-benefit analysis function are shown in figures-2 and 3 w.r.t. α_1 (constant failure rate of subsystem-A) for various values of β_1 (repair rate of subsystem-A) while fixed values of other parameters are $\lambda_1 = 0.07$, $\lambda_2 = 0.06$, $\beta_2 = 0.8$, $\beta_3 = 0.4$, $\theta = 0.8$, $a = 0.5$, $b = 0.5$, $\alpha_2 = 0.11$, $\alpha_3 = 0.12$, $K_0 = 40000$, $K_1 = 5000$, $K_2 = 1000$, $K_3 = 2000$, $K_4 = 1000$ and $K_5 = 5000$.

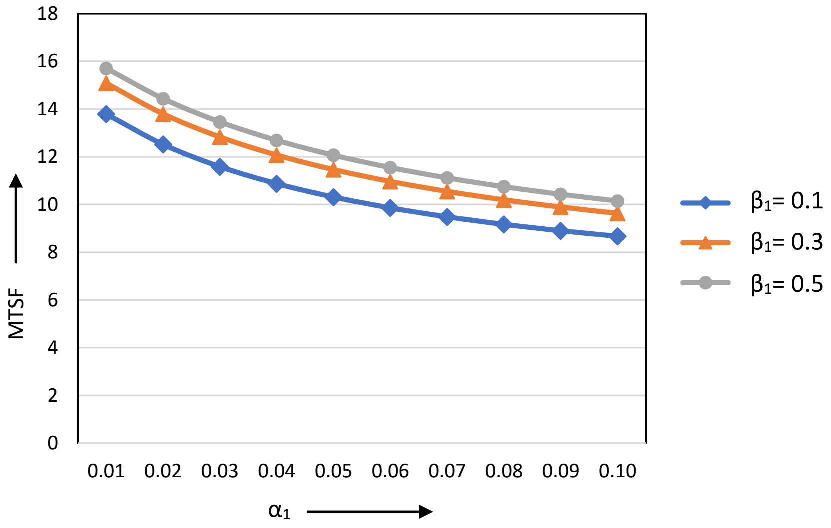

Figure 2: MTSF Vs Failure Rate of Subsystem-A (α_1) for Different Repair Rates

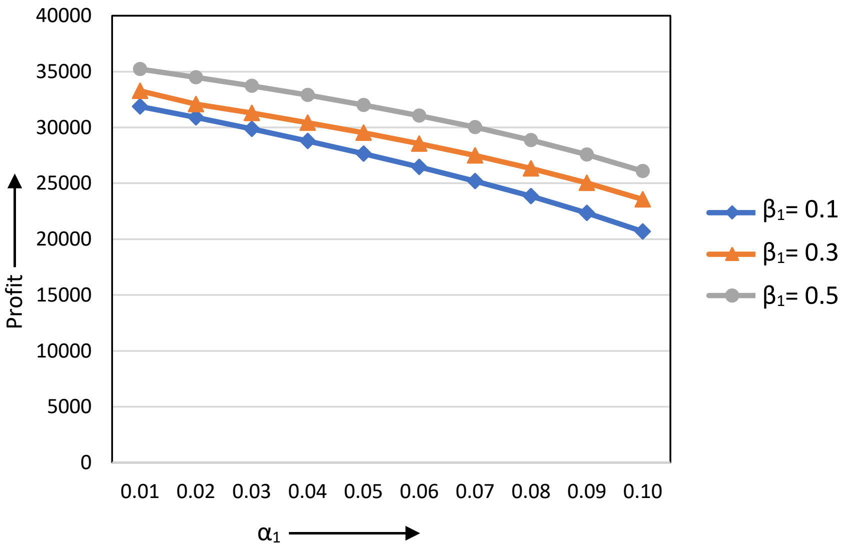

Figure 3: Profit Vs Failure Rate of Subsystem-A (α_1) for Different Repair Rates

9. Conclusion

Figure-2 reveals the behaviour of MTSF for different repair rates of subsystem-A i.e., β_1 (= 0.1, 0.3, 0.5). The figure clearly indicates that MTSF of the system model is decreasing with increasing α_1 and is maximum when repair rate of subsystem-A is maximum. From figure-3, we find that profit of the system model is decreasing with increasing α_1 and is maximum when β_1 is maximum. Therefore, we can conclude that both MTSF and profit of the system model can be optimized by increasing repair rate of subsystem-A.

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