A Comparative Lens on Econometric Standards and Fusion-Based Models

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Abstract: A clear understanding and subsequent prediction of volatility has become a topic of paramount importance for investors, policy makers and market regulators in financial markets. The said understanding and prediction of volatility enables the investors to take informed decisions and reducing risk exposures. Thus said, this study aims to estimate volatility in the IT enabled services industry, which plays an important role in security markets. The methodology of comparative approach between traditional models and a newly blended model named as fuse model has been applied to assess volatility for effective risk management and guided investment decisions for investors.

The methodology collects information on the historical share prices of ITES companies with a special focus on HCL Technologies listed on Indian stock exchanges. This research work delves into the comparative approach between traditional models and fuse models which may be termed as a blended model. The objective of this study approaches towards the concept of best suited model for ITES industry by using four different fuse models namely being: 1. LSTM in conjunction with Fuzzy Logic, 2. Stochastic Process (Markov Decision Process) in conjunction with Fuzzy Logic, 3. Denoising the discrete time series with Discrete Fourier Transform (DFT) followed by Inverse Fourier Transform to obtain the denoised time series which can be treated as an input to LSTM or Time Series Model and finally 4. Ensemble Learning. It is worth mentioning that this type of study is It's a first attempt that this research advocates for a paradigm shift in volatility estimation practices within the Indian ITES sector

Keywords: Volatility, ITES, ARCH family models, LSTM, Fuzzy Logic, MDV, DFT, Ensemble Learning, Fuse Models.

1. Introduction

In the view of dynamic landscape of randomness in the global economy, the Information Technology Enabled Services (ITES) sector stands out as a fast-paced catalyst for innovation, efficiency, and growth. In the light of the profound impact of ITES across industry verticals and other facets of daily life, it has become imperative for investors, policy makers and other stake holders to clearly understand the volatility within the ITES sector. As regards to the Indian economy, where ITES plays a pivotal role, assessing and predicting its volatility becomes even more necessary and critical. The objective of this study is to embark on an empirical journey to delve into the volatility of the ITES sector within the Indian stock market. The aim of the study is to provide valuable insights into the fluctuations and dynamics of this sector by virtue of applying various econometric models. Volatility, as a measure of the dispersion of returns for a given security or market index, serves as a fundamental metric for risk assessment and investment decision-making. A thorough understanding the factors influencing ITES volatility can guide investors in devising robust strategies and also enable the policymakers in formulating effective regulations to foster a stable and conducive economic environment. It is to be noted with pride that over the years, the Indian ITES sector has witnessed an exponential growth coupled with rapid evolution where the key drivers ~~are driven by~~ has been technological advancements, globalization, and randomly changing consumer behaviours. Starting from mere software development with IT consulting then steadily upgrading to business process outsourcing (BPO) and knowledge process outsourcing (KPO) and up to the level of SAAS (Software as a Service), the sector encompasses an ever-increasing ~~wide~~ array of services, each with its unique characteristics and market dynamics. In the context of the Indian Stock market, this immense diversity necessitates a nuanced analysis of volatility, considering the underlying factors specific to each segment and their interplay with broader market forces. Econometric modelling offers a systematic framework for dissecting the complexities of volatility and uncovering the underlying drivers. Through the process of leveraging historical data and statistical techniques, we endeavour to discern patterns, relationships, and causality within the ITES sector. This empirical approach seeks to contribute to the existing body of knowledge surrounding ITES volatility and strive to provide practical implications for investors, policymakers, and industry stakeholders.

The study is structured as follows:

Following this introduction, a comprehensive review of relevant literature, highlighting previous research endeavours and theoretical foundations are provided. Subsequently, we outline the methodology employed in our empirical analysis wherein the data sources, variables are elucidated followed by a comparative analysis between econometric techniques and the newly blended method named as fuse model. On completion of the comparative analysis, we present our findings, accompanied by detailed discussions and interpretations. Finally, we offer concluding remarks, summarizing key insights and avenues for future research. In essence, this study endeavours to shed light on the volatility of ITES within the Indian stock market with HCL Technologies stock price, offering valuable perspectives for stakeholders navigating the dynamic landscape of the global economy. It needs to be mentioned that understanding the volatility of the ITES sector is crucial due to its significant impact on economic growth, employment, and innovation. The Indian ITES industry, since its inception ~~in particular~~, has emerged as a global leader, contributing substantially to the country's GDP and employment generation. However, the sector is not immune to market fluctuations and external shocks, which can have far-reaching implications for businesses, investors, and policymakers. We, by analysing the volatility of ITES stocks in the Indian market, aim to provide insights that can empower informed investment decisions, risk management strategies, and policy interventions. Through the application of econometric models, we seek to identify the key determinants of ITES volatility, including internal factors such as firm-level characteristics and external factors such as market sentiment and macroeconomic conditions.

ARCH Family models as traditional model and four different fuse models which are LSTM in conjunction with Fuzzy Logic, Stochastic Process (Markov Decision Process) in conjunction with Fuzzy Logic, Denoising the discrete time series with Discrete Fourier Transform (DFT) followed by Inverse Fourier Transform to obtain the denoised time series which can be treated as an input to LSTM or Time Series Model and finally Ensemble Learning have been used to guesstimate the Volatility. HCL Technologies one of n the biggest ITES company has been taken into consideration from Indian Stock Market for further analysis. The data has been collected from Yahoo Finance from April 2013 to September 2024 for analysis.

2. Literature Survey

Several studies have investigated volatility estimation in the Indian stock market, with a specific focus on ITES companies. "Ali, F., Suri, P., Kaur, T., & Bisht, D. (2022) employed the GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model to estimate volatility in a sample of ITES firms listed on the National Stock Exchange of India (NSE)" [1]. Their findings revealed significant volatility

clustering and persistence in ITES stock returns, indicating the presence of time-varying volatility dynamics in the sector.
In creating a long-term study on financial market volatility finding procedures, Poon and Granger (2003) compared different forms of models like machine learning and econometrics [4]. Banerjee and Mukherjee (2015) showed the clustering of volatilities across Indian markets using GARCH and TGARCH, showing same may be handy for sector-wise investigations [12].
Choudhry (2005) examined volatility spill-overs in the Asian stock markets toward the greater understanding of how volatility spills over into the ITES industry in India [5]. Majumdar and Basu (2019) analyzed the macroeconomic determinants of volatility on IT stocks, noticing the currency rate and global demand as two external factors [10].
Such fusion will result in hybrid models in which deficiencies related to econometric procedures can be corrected with their intelligence or learning. GARCH models were being compared to neural networks by Gupta and Modak (2020) and proved neural networks to be better over the previous in non-linear environments [7]. Zhang (2003) takes retrospective review of neuro-fuzzy hybrid learning models for time-series forecasting that underperform while paving the way to new model development [14].
Patel and Patel (2019) have analyzed the volatility of the Indian IT sector during global events by using TGARCH models, with a special focus on exogenous shocks [13]. Nayak and Pattnaik (2021) explored fuzzy logic-based models for volatility prediction, focusing on their capability to manage uncertainty and imprecision [11]. Choudhury and Misra (2020) compared the performance of machine learning models with traditional models, where the fused model showed efficiency in the prediction process [16]. Kumar and Sharma (2019) gave an all-inclusive comparison of the econometric and neural network models, which revealed the strengths and weaknesses of the traditional versus modern approaches in estimating volatility in the Indian ITES sector [9].
The literature indicates that while the traditional econometric models such as ARCH and GARCH can also be of help, the hybrid ones with embedded AI and fuzzy logic do better in the dynamic ITES industry. Future work should extend the range of hybrid methods to account for the non-linearity and handle challenging datasets.
The hybrid GARCH-ANN model proposed by Jha and Das (2017) typically has the best prediction accuracy for Indian markets [8]. Engle and Rangel (2008) developed the spline-GARCH models for smoothing out the low-frequency variations in volatility, which can be applied to sector-specific usage of ITES industry[6]. Yu and Meyer (2006) consolidated and proffers the method to be employed in comparison with the conventional econometrics methods in advanced stochastic volatility models [15].
In a similar vein, “Gupta et al. (2020) conducted a comparative analysis of volatility estimation techniques for ITES stocks, including GARCH, EGARCH (Exponential GARCH), and TGARCH (Threshold GARCH) models” [7]. Using daily stock price data from the Bombay Stock Exchange (BSE), they found that the EGARCH model outperformed other models in capturing the asymmetric volatility patterns observed in ITES stock returns.
Furthermore, “Ali, F., Suri, P., Kaur, T., & Bisht, D. (2022) explored the impact of macroeconomic factors on the volatility of ITES stocks in the Indian market [1]. Employing a multivariate GARCH framework, they examined the effects of variables such as exchange rates, interest rates, and GDP growth on ITES stock volatility”. Their results indicated that macroeconomic factors significantly influence the volatility dynamics of ITES companies, highlighting the interconnectedness between the ITES sector and the broader economy.
Other empirical studies have focused on specific aspects of volatility estimation in ITES companies. For instance, “Gao et al. (2022) investigated the role of investor sentiment in driving volatility in ITES stocks s[3]. Using sentiment analysis techniques on social media data, they demonstrated that investor sentiment has a significant impact on the volatility of ITES companies, particularly during periods of market uncertainty”.
But there is very less work with Fuzzy Integration and literally none found with fuse model.

Objective

The objective of this effort is to arrive at an empirical thumb rule which enables in the decision-making process of an investor. To strive for the objectives, the following questions needs to be answered:

1. Apart from conventional Time-Series analysis are there any other alternatives that dives deep into the fine sentimental or human perspectives of the stock market or about a particular stock?
2. In the event of availability of such alternatives, are those alternatives able to address the randomness within the system and hence can give a guidance to the investors?

3. Methodology

In the endeavour of thriving for the conceptually and technically feasible answers to the above questions the following Models may be considered:

a. LSTM in conjunction with Fuzzy Logic.

b. Stochastic Process (Markov Decision Process) in conjunction with Fuzzy Logic.

c. Denoising the discrete time series with Discrete Fourier Transform (DFT) followed by Inverse Fourier Transform to obtain the denoised time series which can be treated as an input to LSTM or Time Series Model.

d. Ensemble Learning.

Historical price data for a selected set of stocks from the Indian Equity market has been collected. Several econometric models were employed to estimate volatility in ITES stocks, with a focus on capturing time-varying volatility patterns and asymmetries in stock returns. The primary models used in the analysis include:

'Generalized Autoregressive Conditional Heteroskedasticity (GARCH)' Models: GARCH models are widely used for modelling volatility dynamics in financial time series data. The basic GARCH model specifies that volatility is a function of lagged squared residuals, capturing the persistence and clustering of volatility. GARCH model have some division as EGARCH, TGACH, FGARCH, FIEGARCH, TARCH, PARCH.

The parameters of the econometric models were estimated using maximum likelihood estimation techniques. The adequacy of the model specifications was assessed using diagnostic tests such as the Ljung-Box test for autocorrelation in residuals and the 'ARCH-LM' test for ARCH effects.

To ensure the robustness of the results Fuse models are used which are discussed as hereunder:

a.) Fuzzy Logic is an extension of classical logic that allows for reasoning about imprecise or uncertain information. In traditional logic, propositions are either true (1) or false (0). In contrast, fuzzy logic allows propositions to have a degree of truth ranging between 0 and 1, reflecting the real-world situations where boundaries between categories are not always clear-cut. In fuzzy logic, variables are often described using linguistic terms (e.g., "high", "low", "medium"). Each linguistic term corresponds to a fuzzy set. As an example, in fuzzy logic the volatility of a stock may be defined as High, Low or Stable.

Proceeding with this concept the following steps may be followed:

1. Defining the Member: Here we can use the most prevalent Triangular Membership function. Let the vertices of the Membership triangle be considered as a, b, c. Then the Triangular Membership function can be described by the following inequalities:

A membership function $\mu_A(x)$ assigns a membership value (degree of truth) to each element of the fuzzy set.

$$\mu_A(x) = f(x) = \begin{cases} 0, & \text{if } x \le a \\ \frac{x-a}{x-b}, & \text{if } a \le x \le b \\ \frac{c-x}{c-b}, & \text{if } b \le x \le c \\ 0, & \text{if } x \ge c \end{cases} \qquad \text{---Equation(1)}$$

In this case of stock market, the volatility of the stock may be considered to be within range of 0 to 15% and the parameters a, b, c may be assigned as per the class of Volatility as follows:

1. The low volatility range can be from 0 to 5% with peak at 2.5% (i.e a=0, b=2.5 and c=5)

2. The Medium Volatility range can be from greater than 5% to 10% with Peak at 7.5% (i.e a=2.5, b=7.5 and c=10)

3. The High Volatility range can be from greater than 10% to 15% with peak at 12.5%. (i.e a=7.5, b=10 and c=15)

Replacing the values of a, b, c for each volatility class we can calculate the following:

$\mu_{LOW}(x)$ = {0 for x<=a
(x-2.5) / (2.5) for 0<=x<=2.5
(5-x)/ (2.5) for 2.5<=x<=5 (1)
0 for x>= 5}

$\mu_{MEDIUM}(x)$ = {0 for x<=2.5
(x-2.5) / (5) for 2.5<=x<=7.5
(10-x)/ (2.5) for 7.5<=x<=10 (2)

0 for x>= 10}

$\mu_{HIGH}(x)$ = {0 for x<=7.5

(x-7.5) / (2.5) for 7.5<=x<=10 (3)

(15-x)/ (5) for 10<=x<=15

0 for x>= 15}

As an example, let us take the value of x = 9 for a particular ITES stock. Now we need to calculate the degree of membership of the said stock for each of the categories. Replacing the value of x as equal to 9 we get:

a. $\mu_{LOW}(x)=0$
b. $\mu_{MEDIUM}(x) = 0.4$
c. $\mu_{HIGH}(x) = (20-x)/ (2.5) = 0.6$

Hence it is clear from the above values that the particular ITES stock falls under High Volatility. The values of μ_{LOW}, μ_{MEDIUM} and μ_{HIGH} will be crucial in the calculation for forecasting of volatility.

b.) Discrete Fourier Transform

In time-series data, such as stock volatility etc., data can often be noisy due to sudden market changes, trading anomalies, or random fluctuations. This noise can make the underlying trends and patterns obscure which are critical for analysis or predictions. The Fourier Transform allows us to transform the series from a time-domain to the frequency domain by decomposing the time-series data into its constituent frequency components and thus making it possible to identify and filter out high-frequency noise while preserving the core signal.

In financial time-series analysis, Fourier Transform can be used for:

- Decomposing volatility into different frequency components to identify periodic patterns.
- Filtering noise from stock data by focusing on specific frequencies.
- Detecting seasonality and cycles, such as daily, weekly, or monthly trends.
- Predicting future trends by analysing frequency patterns in historical data.

For discrete data, such as daily stock prices, the Discrete Fourier Transform (DFT) is used:

$$X[k] = \sum_{K=0}^{N-1} x[n]e^{-i2\pi kn/N} \quad \text{----Equation (2)}$$

Where:

- x[n] is the time-series data (e.g., stock prices or volatility values),
- X[k] represents the amplitude of the k-th frequency component,
- N is the number of data points.

In AI/ML, Fast Fourier Transform (FFT) algorithms are commonly used to efficiently compute the DFT of large datasets.

After obtaining the DFTs, effective filtering can be used to discard the low frequency values by equating them to zero. Once this is done, Inverse DFT is applied to get the denoised Time-Series which will be a smother curve and can be used for detailed analysis and prediction.

The inverse DFT is given by:

$$X[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]e^{i2\pi kn/N} \quad \text{----Equation (3)}$$

Let us now consider the following example:

Given stock volatility data for 10 days:

X[n]=[12,15,14,13,10,12,11,13,12,14]

Applying DFT to the above data we get:

Table 1: DFT

k	X[k] MAGNITUDE
0	126
1	6.5
2	3.5
3	2.8
4	1.5
5	0.5
6	0.4
7	0.3
8	0.2
9	0.1

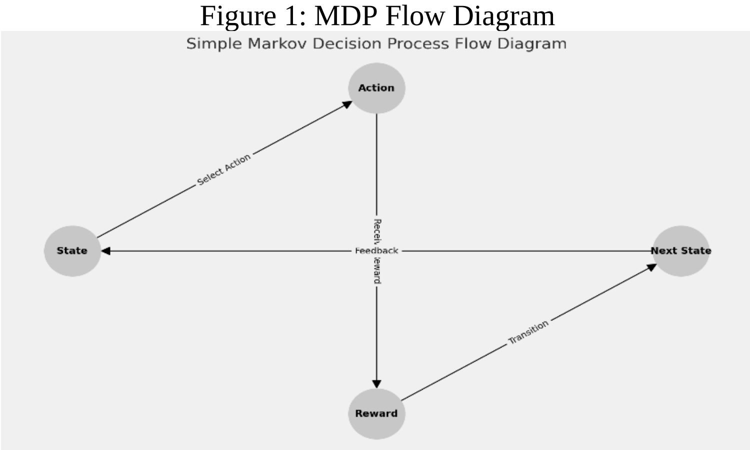
It is amply clear from the above frequency table that for k= 0 to 4, the frequencies contribute to the series whereas for k= 5 to 9, the frequencies can be considered as noise and hence can be equated to zero. Performing inverse DFT we get the denoised series as follows:
X[n] = [14.03,13.5,13.2,12.9,12.8,13.0,13.2,13.1,13.4,14.0]
Based on this Denoised Series, requisite analysis and predictions may be conducted.
c.) Markov Decision Process (MDP)
It is a fact that in financial markets volatility estimation undoubtedly plays a crucial role in risk management and investment decision-making. Accurate volatility prediction can help in assessing potential market risks and guide trading strategies. Assuming that volatility being a random variable, this option aims to apply Markov Decision Processes (MDP) to estimate and model volatility specifically within the IT-enabled services (ITES) industry. MDPs are highly suitable for sequential decision-making problems where outcomes are uncertain, making them ideal for capturing volatility patterns influenced by changing market conditions.
A Markov Decision Process (MDP) is a mathematical framework used to model decisions in stochastic environments. It provides a structured way to evaluate a sequence of actions and their potential outcomes, considering the inherent uncertainty in the decision process. MDPs consist of the following components:

1. States (S): Possible states that represent market conditions or levels of volatility.
2. Actions (A): Decisions or trading actions (e.g., buy, hold, sell) that can impact volatility.
3. Transition Probabilities (P (s′| s, a)): Probability of transitioning to a new state s′ given the current state s and action a.
4. Rewards (R (s, a)): Expected return or loss when taking action, a in state s.
5. Policy (π(s)\pi(s)): Strategy that defines the action to take in each state to optimize expected returns.

The core equation of a Markov Decision Process is based on the Bellman equation for the value function:
$V^{\pi}(s) = \sum_{a \in A} \pi(a|s) \sum_{s' \in S} P(s'|s, a)[R(s, a, s') + \gamma V^{\pi}(s')]$ -----Equation (4)
Where:

- $V^{\pi}(s)$: Value of state, s under policy π.
- π(a|s): Probability of taking action a in state, s under policy π.
- P(s′|s,a): Transition probability from state, s to state, s′ given action a.
- R(s,a,s′):: Reward received when transitioning from s to s′ via a.
- γ: Discount factor (between 0 and 1).

Figure 1: MDP Flow Diagram



The goal of MDP in volatility estimation is to determine an optimal policy π\pi that minimizes the risk (volatility) while maximizing the returns over a certain period.
d.) Markov Decision Process (MDP) In Conjunction with Fuzzy Logic
In this context of volatility estimation, a hybrid model of MDP in conjunction with Fuzzy Logic may be used. MDP is suitable as the stock prices or stock volatility are completely random and hence can be considered as a stochastic process which can be modelled using MDP. Fuzzy Logic may be used to fine tune the model for better accuracy as Fuzzy Logic is capable of modelling the intricate uncertainties in the data. The broad methodology of this Hybrid Model has been entailed below:
Step 1: Data Collection

Collect historical data on stock prices, trading volumes, and financial news related to companies in the ITES sector.
Step 2: Fuzzification
Apply fuzzy logic to convert the raw input data into fuzzy sets. For example, price fluctuations can be converted into fuzzy membership degrees of low, medium, and high volatility.
Step 3: Define MDP Components

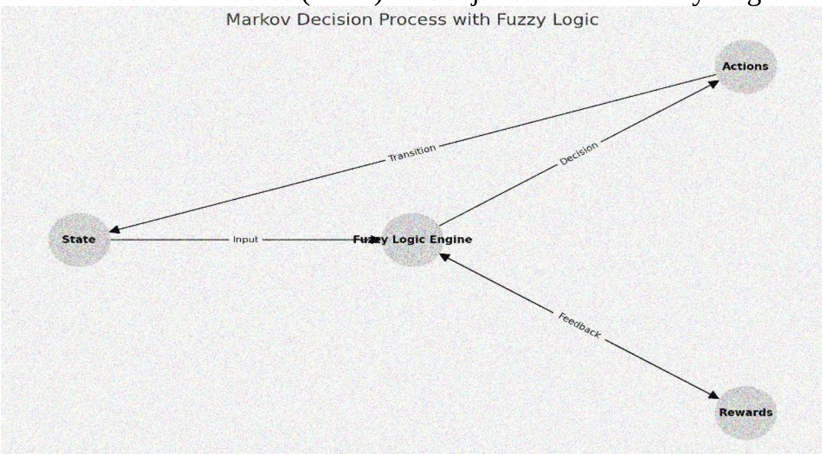
- States: Represent different levels of volatility (low, medium, high).
- Actions: Define possible actions (buy, hold, sell).
- Transition Probabilities: Calculate the probabilities of moving between different volatility states based on historical data.
- Rewards: Calculate the expected return for each action in each state.

Step 4: Apply Bellman Equation
Use the Bellman equation to estimate the optimal value of each state and action pair, and determine the best policy (i.e., the action that maximizes future expected rewards).
Using MDP's Bellman equation the following function is created:

$$V^{\pi}(s) = \sum_{a\in A} \pi(a|s) \sum_{s'\epsilon S} \mu p(s'|s\,,a)[\mu g(s,a,s') + \gamma V^{\pi}(s')]$$ ---------Equation (5)

μ_P and μ_R are the fuzzy transition probability and reward, respectively.
And the flow diagram is given as hereunder:

Figure 2: Markov Decision Process (MDP) In Conjunction with Fuzzy Logic flow diagram



Step 5: Risk Management
Estimate the volatility of stock prices using the hybrid MDP-fuzzy logic model. This will allow the investors to assess risk more effectively.
An elaborate explanation of step 3 has been enumerated below:
Use Fuzzy Logic to classify volatility into states (e.g., low, moderate, high).

1. Fuzzy Transition Probabilities Between States: We calculate the probability of moving from one state to another based on historical volatility trends
2. Define Actions and Rewards for Each Fuzzy State: In each state, we have three actions: Increase Position (A1), Hold Position (A2), and Decrease Position (A3). Rewards are based on expected returns
3. Value Function Calculation for Volatility (S1, S2, S3) Using Bellman Equation
4. Iterative Policy Optimization Using MDP with Fuzzy Logic
5. Monte Carlo Simulations to Validate Model Performance: Run simulations across volatility states using the fuzzy MDP model to validate the policy's effectiveness under various market conditions.
6. Sensitivity Analysis on Fuzzy Transition Probabilities: Test the model's robustness by varying transition probabilities slightly and examining how the optimal policy changes, helping adapt the model for changing market conditions.
7. Risk Assessment for Investment Decision-Making: Using the optimal policy, assess the risk and potential rewards associated with each action in each volatility state. This helps guide real-time decision-making in volatile ITES markets.

e.) LSTM In Conjunction with Fuzzy Logic
LSTM networks excel at analysing sequential data, making them suitable for modelling the complex, time-dependent behaviour of stock price volatility. Fuzzy Logic, on the other hand, allows for handling the uncertainty and imprecision that often characterize financial data. Together, these methods form a robust framework for predicting and understanding volatility trends in the ITES industry.

LSTM Recurrent Relations is defined as:

Forget GATE:

$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$ -------- Equation (6)

Where: f_t is the forget gate, W_f is the weight matrix for the forget gate, h_{t-1} is the previous hidden state, x_t is the current input, b_f is the bias term, σ is the sigmoid activation function.

Candidate Memory Cell:

$\tilde{C}_t = \tanh(W_c \cdot [h_{t-1}, x_t] + b_c)$ ------- Equation (7)

Memory cell update:

$C_t = f_t . C_{t-1} + i_t . \tilde{C_t}$ ------- Equation (8)

Output gate:

$o_t = \sigma(W_o . [h_{t-1}, x_t] + b_0)$ -------- Equation (9)

Hidden state update:

$h_t = o_t . \tanh(C_t)$ -------- Equation (10)

Let's denote a fuzzy input variable A (e.g., a sequence input or a previous hidden state), which is fuzzified into a fuzzy set A_{fuzzy}. The fuzzy output can be applied to adjust the LSTM gate values or the memory update process.

Fuzzification: $A_{fuzzy} = \mu_A(x_i)$ ------ Equation (11)

where $\mu_A(x_t)$ is the membership function

Defuzzification: Once the fuzzy output has been calculated (e.g., the new forget gate), we convert the fuzzy values back to crisp values to be used in the LSTM equations. The defuzzified value f_t can be computed using methods like the centroid method:

$f_t = \frac{\sum(\mu_A(x_i).A)}{\sum \mu_A(x_i)}$ ----- Equation (12)

Combining LSTM with Fuzzy Logic:

Adjusted Forget Gate:

$f_t = \sigma(W_f . [h_{t-1}, x_t] + b_f) + \alpha . \mu_{High}(x_t)$ ----- Equation (13)

Adjusted Hidden state:

$h_t = o_t . \tanh(C_t) + \beta . \mu_{Low}(x_t)$ ----- Equation (14)

Where $\mu_{Low}(x_t)$ represents the fuzzy logic adjustment for low input values, and β is a tuning parameter.

Methodology

Step 1: Data Collection and Preprocessing

1. Collect daily price data for ITES sector stocks, calculate returns, and measure historical volatility.
2. Split data into training, validation, and test sets.

Step 2: Define Fuzzy Logic-Based Volatility States

Define fuzzy categories of volatility based on the daily return data:

- Low Volatility: Below 1% daily return.
- Moderate Volatility: 1% to 2% daily return.
- High Volatility: Above 2% daily return.

Step 3: Train the LSTM Model

1. Use the processed data to train an LSTM model to forecast future volatility based on past price movements.
2. Integrate the LSTM's outputs with Fuzzy Logic to generate linguistic volatility states.

Step 4: Define Fuzzy Rules and Decision-Making

Use Fuzzy Logic rules to make investment decisions based on the LSTM model's volatility forecast. Examples:

- If volatility is predicted to be high, then decrease investment.
- If volatility is predicted to be low, then increase investment.

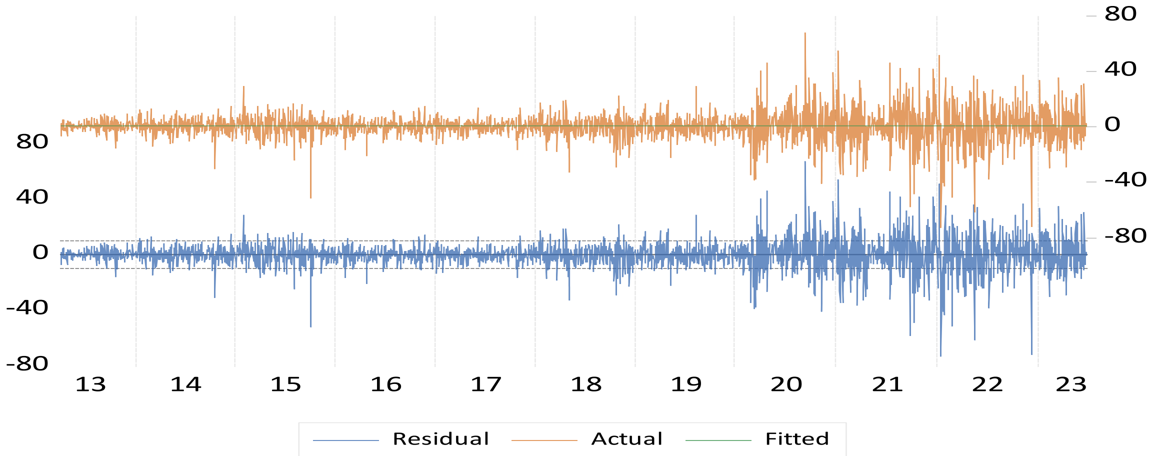
Figure 3: LSTM In Conjunction with Fuzzy Logic block diagram

And finally, ensemble machine learning.
The best fit model are tested based on RMSE, MAE, MAD and MAPE.

4. Results and Analysis

ARCH Family Models for HCL Technologies.
After fitting the regression model with constant, we get the residuals as in the below mentioned diagram:

Figure 5: Residuals after fitting regression model



‘From the above diagram it has been found that the periods of low volatility are tend to be followed by prolonged period of low volatility and same for the high volatility. When this happens, we have all the justifications to run the ARCH family model for volatility guesstimation. But we should cross validate the same by ARCH test that whether we should run the ARCH family model or not’. The ‘ARCH Test result’ is as follows: -

Table 1

Heteroskedasticity Test: ARCH

F-statistic	53.17703	Prob. F(1,2521)	0.0000
Obs*R-squared	52.11982	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 02/21/24 Time: 19:32
Sample (adjusted): 4/03/2013 6/28/2023
Included observations: 2523 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	88.93495	6.431286	13.82849	0.0000

RESID^2(-1)	0.143729	0.019710	7.292258	0.0000

R-squared	0.020658	Mean dependent var	103.8631
Adjusted R-squared	0.020269	S.D. dependent var	309.3897
S.E. of regression	306.2381	Akaike info criterion	14.28740
Sum squared resid	2.36E+08	Schwarz criterion	14.29202
Log likelihood	-18021.55	Hannan-Quinn criter.	14.28907
F-statistic	53.17703	Durbin-Watson stat	2.027413
Prob(F-statistic)	0.000000		

‘From the above test result we can see that p value is 0 and is less than 0.05 i.e 5 percent. So, we can reject null hypothesis and accept the alternative hypothesis. The null and alternative hypothesis is as follows: -

“Null Hypothesis: There is no ARCH effect”

“Alternative Hypothesis: There is ARCH effect”

“So, we can go for ARCH family models”.

After using the ARCH family models, it was found that FIEGARCH model is the best fit model with lowest AIC and SIC value.

The result is as hereunder:

FIEGARCH (1, 1) model

The model result is shown in the following table:

Table 2

Dependent Variable: DHCLTECH
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)

Sample (adjusted): 4/02/2013 6/28/2023
Included observations: 2524 after adjustments
Failure to improve likelihood (non-zero gradients) after 144 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.353298	0.124466	2.838517	0.0045
	Variance Equation			
OMEGA	3.387205	0.029366	115.3450	0.0000
ALPHA	-1.004301	6.54E-05	-15352.75	0.0000
BETA	0.996808	1.54E-05	64759.07	0.0000
THETA1	0.262190	0.020378	12.86600	0.0000
THETA2	-0.003589	0.011623	-0.308767	0.7575
D	0.567112	0.103450	5.481968	0.0000

R-squared	-0.000022	Mean dependent var	0.401291
Adjusted R-squared	-0.000022	S.D. dependent var	10.19135
S.E. of regression	10.19146	Akaike info criterion	6.994823
Sum squared resid	262053.5	Schwarz criterion	7.011002
Log likelihood	-8820.467	Hannan-Quinn criter.	7.000694

The AIC and SIC values are 6.99 and 7.01.

From the above analysis the minimum value of AIC and SIC is for FIEGARCH (1,1) model which is 6.99 and 7.01. So as per criterion the best model fitted is FIEGARCH (1, 1).

Though the FIEGARCH (1, 1) model is the best model but RMSE: 0.06401286385503023 MAE: 0.04953527590649164, MAPE: 24.809987954642395%, we must go for diagnostic checking. So, we must estimate the model firstly by checking whether there is any serial correlation present in the model?

Let’s start with correlogram of squared residuals. The results are taken with 36 lags which are displayed below.

Table 3

Sample (adjusted): 4/02/2013 6/28/2023
Included observations: 2524 after adjustments

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
| |	| |	1	0.006	0.006	0.0870	0.768
| |	| |	2	-0.014	-0.015	0.6181	0.734
| |	| |	3	-0.006	-0.006	0.7053	0.872
| |	| |	4	0.011	0.011	1.0351	0.904
| |	| |	5	0.007	0.007	1.1753	0.947
| |	| |	6	-0.017	-0.016	1.8692	0.931
| |	| |	7	0.010	0.010	2.0996	0.954
| |	| |	8	0.028	0.027	4.0720	0.851
| |	| |	9	-0.005	-0.005	4.1234	0.903
| |	| |	10	-0.019	-0.018	5.0555	0.887

*Probabilities may not be valid for this equation specification.

From the above result we got Q Statistics and the respective probability value. The null and alternative hypothesis are: -
Null Hypothesis: There is no serial correlation in the residuals
Alternative Hypothesis: There is serial correlation in the residuals.
The above p values shows that null hypothesis is true as p>0.05 for all Q statistics. So, there is no serial correlation.
Next, we have to check whether there is ARCH effect or not. For that ARCH-LM test is performed for which the result is as follows: -

Table 4

Heteroskedasticity Test: ARCH

F-statistic	0.086816	Prob. F(1,2521)	0.7683
Obs*R-squared	0.086882	Prob. Chi-Square(1)	0.7682

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 02/21/24 Time: 21:20
Sample (adjusted): 4/03/2013 6/28/2023
Included observations: 2523 after adjustments

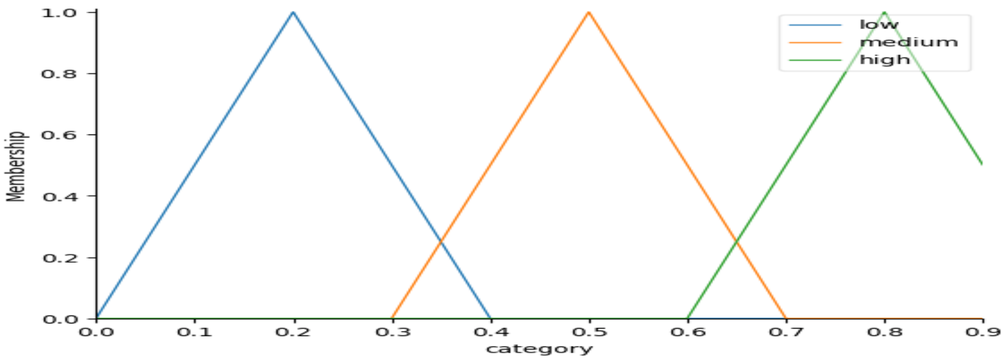
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.018988	0.051503	19.78519	0.0000
WGT_RESID^2(-1)	0.005868	0.019916	0.294646	0.7683

R-squared	0.000034	Mean dependent var	1.025004
Adjusted R-squared	-0.000362	S.D. dependent var	2.374603
S.E. of regression	2.375033	Akaike info criterion	4.568692
Sum squared resid	14220.41	Schwarz criterion	4.573316
Log likelihood	-5761.405	Hannan-Quinn criter.	4.570370
F-statistic	0.086816	Durbin-Watson stat	1.999762
Prob(F-statistic)	0.768289		

The Null hypothesis and Alternative hypothesis are:-
Null Hypothesis: There is no ARCH effect
Alternative Hypothesis: There is ARCH effect.
As p values are > 0.05 so we have to accept the null hypothesis and conclude that there is no ARCH effect in the model.

The Null Hypothesis: The residuals are normally distributed and the Alternative Hypothesis: The residuals are not normally distributed.
As $p<0.05$ we have to reject the null hypothesis. And accept the alternative hypothesis which is not desirable. But as this model has no serial correlation and ARCH effect so according to economists, we can accept the model.
Finally, from granger causality test it was found that volatility of HCL Technologies granger causes the movement of NIFTY 50 and SENSEX.
The above model is a traditional model. Comparing with the fuse models the analysis is as:
a.) Using LSTM in conjunction with Fuzzy Logic

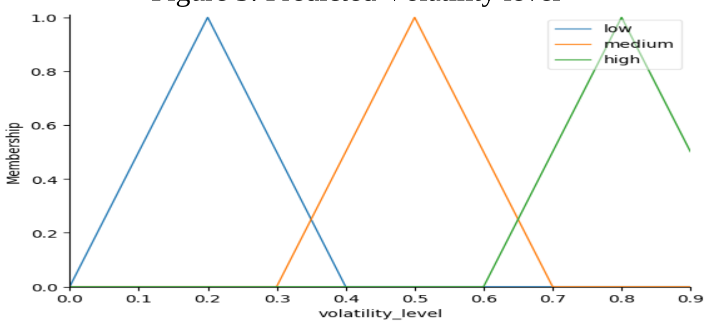
Figure:4 Volatility Category



Predicted Volatility from LSTM: 0.00040889004594646394
Fuzzy Volatility Category: 0.2, RMSE: 0.21326531818616543, MAD: 0.2052904118103504, MAPE: 1437.6262748233046%
b.) Using Stochastic Process (Markov Decision Process) in conjunction with Fuzzy Logic.
Creating Transition matrix as follows:

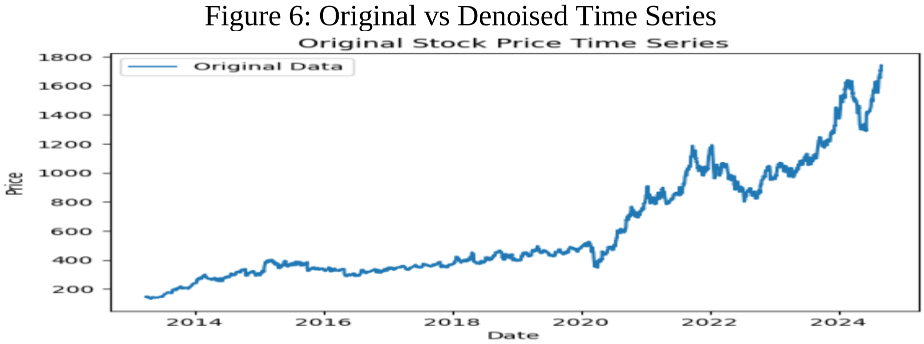
State	Low	Medium	High
Low	1.0	0.0	0.0
Medium	0.0	1.0	0.0
High	0.0	0.0	1.0

Figure 5: Predicted Volatility level



Predicted Volatility Level (fuzzy): 0.5, RMSE: 0.49568833468960805, MAD: 0.4524346471316296, MAPE: 2875.2441271209914%
c.) Denoising the discrete time-series with Discrete Fourier Transform (DFT) followed by Inverse Fourier Transform to obtain the denoised Time Series which can be treated as an input to LSTM or Time Series Model.

Figure 6: Original vs Denoised Time Series



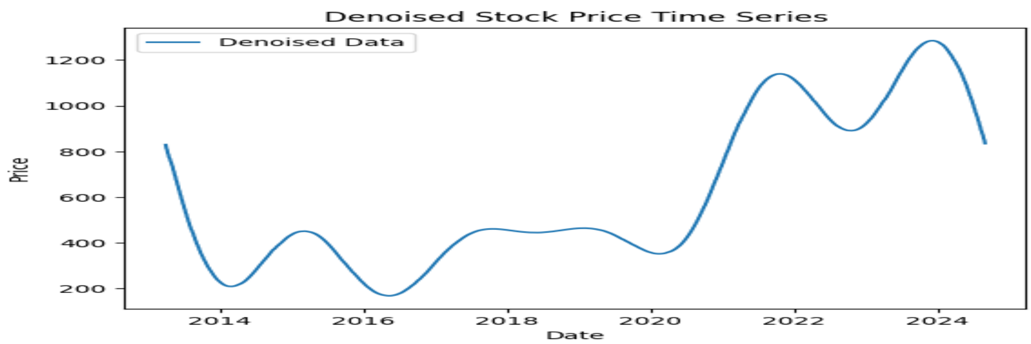
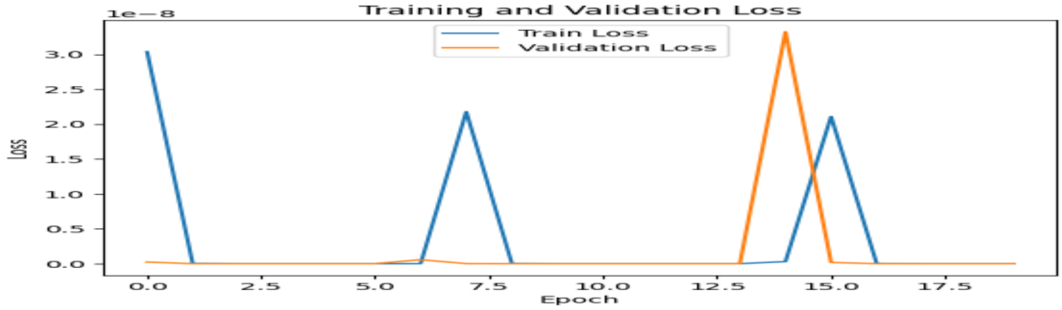
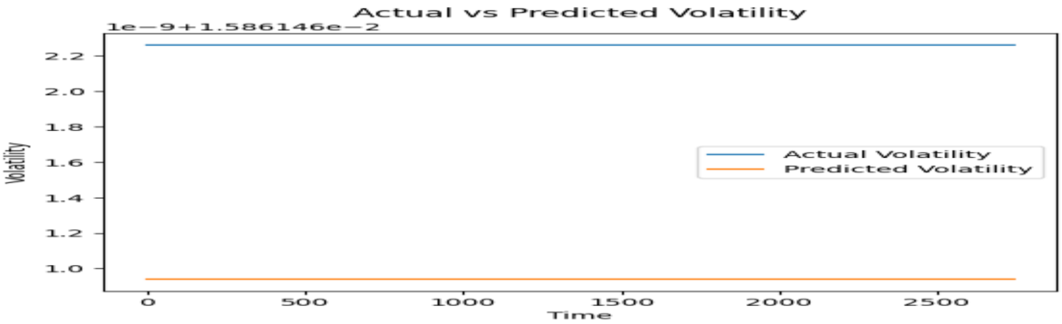


Figure 7: Actual vs Predicted Volatility after training and testing

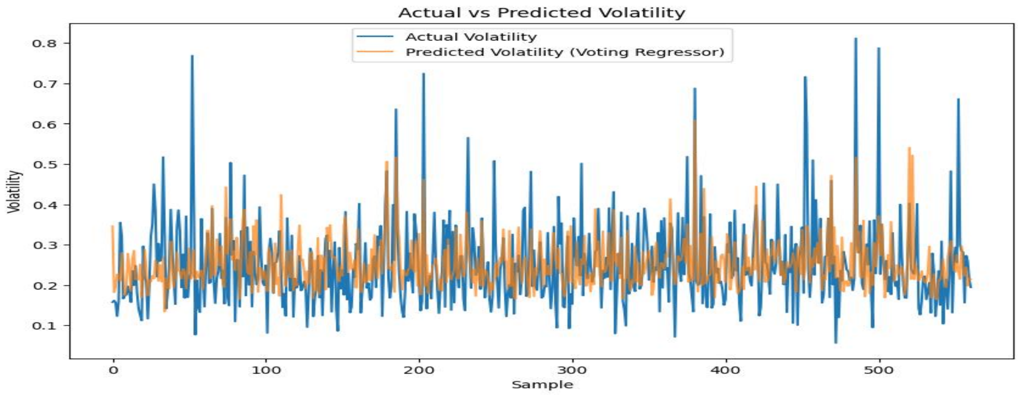




RMSE: 5.410352724198035e-10, MAE: 5.410352724198035e-10, MAPE: 3.411005010311733e-06%

d.) Ensemble Learning:

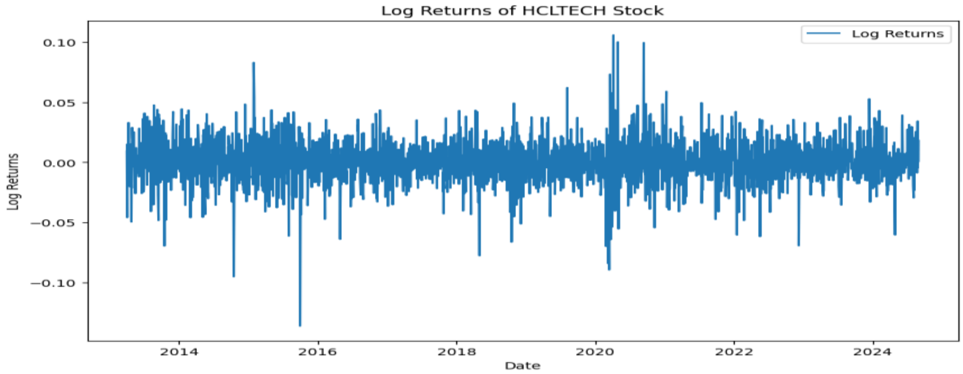
Figure 8: Actual vs Predicted volatility



Random Forest - RMSE: 0.1008, MAE: 0.0744, MAPE: 33.42%
Gradient Boosting - RMSE: 0.0919, MAE: 0.0677, MAPE: 30.92%
Voting Regressor - RMSE: 0.0945, MAE: 0.0697, MAPE: 31.64%

e.) ARIMA – GARCH Blended Model:

Figure 9: Log Returns ARIMA-GARCH Blended Model



Data Scale Warning: y is poorly scaled, which may affect convergence of the optimizer when estimating the model parameters. The scale of y is 0.000296. Parameter
estimation work better when this value is between 1 and 1000. The recommended
rescaling is 100 * y.

Figure 10: Estimated Volatility ARIMA-GARCH Blended Model

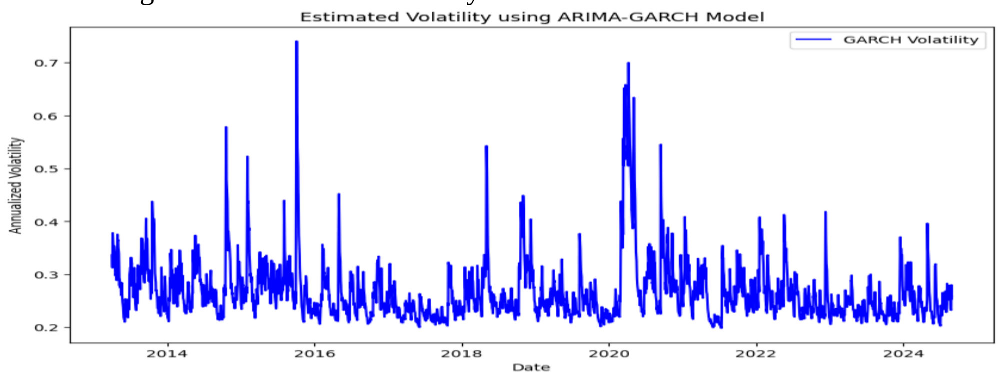
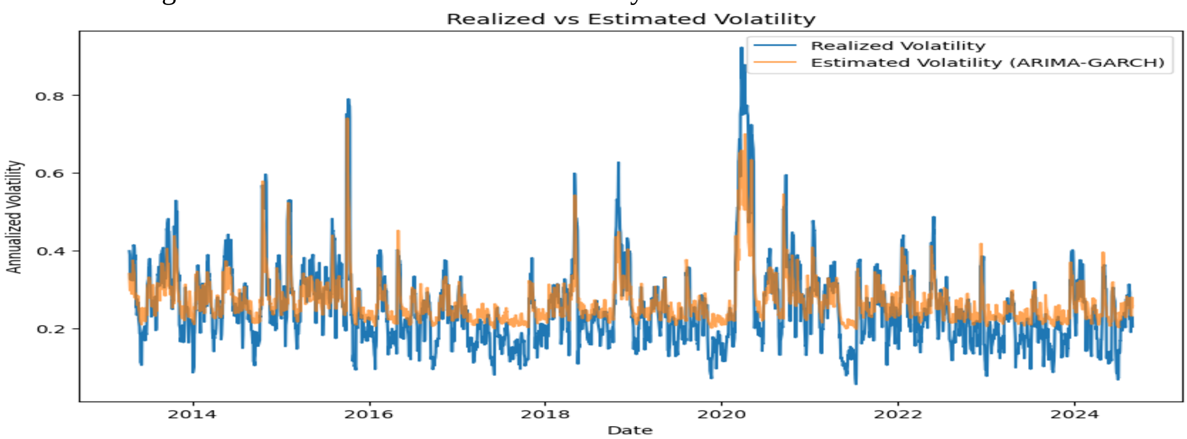


Figure 11: Realised vs Estimated Volatility ARIMA-GARCH Blended Model



RMSE: 0.06499021364318998, MAE: 0.0502332855685317, MAPE: 25.150775285047256%

Based on the above fuse models and traditional models the accuracy is measured with the value of RMSE, MAD, MAE and MAPE values are as hereunder:

Table 5: Model Accuracy

Model	Volatility	RMSE	MAD/MAE	MAPE
FIEGARCH	0.24	0.06	0.05	24.80
LSTM with Fuzzy	0.20	0.21	0.20	1437.62
MDP with Fuzzy	0.5	0.50	0.45	2875.24
DFT with IFT and LSTM*	0.20	$5.41e^{-10}$	$5.41e^{-10}$	$3.411e^{-06}$
Random Forest	0.20	0.1008	0.07	33.42
Gradient Boosting	0.20	0.09	0.06	30.92
Voting regressor	0.20	0.09	0.07	31.64
ARIMA - GARCH	0.24	0.06	0.05	25.15

*The above table reflects that Denoising the discrete time-series with Discrete Fourier Transform (DFT) followed by Inverse Fourier Transform to obtain the denoised Time Series which can be treated as an input to LSTM or Time Series Model is the best fuse model as the RMSE, MAE and MAPE is the best model.

5. Conclusion

In this empirical study, the ARCH family model is used to guesstimate the volatility for HCL Technologies stock price, and the unit root test is utilized to determine whether time series data are stationary. ARCH family models are used to predict the volatility and Granger Causality Test is performed to find out whether the Volatility Index is highly effecting the stock price of HCL technologies or not. Daily data from April 2013 to September 2024 is taken as sample data. By the rule of lowest value of AIC and SIC FIEGARCH (1, 1) model is the best model for traditional model. But the comparative study states that Denoising the discrete time-series with Discrete Fourier Transform (DFT) followed by Inverse Fourier Transform to obtain the denoised Time Series which can be treated as an input to LSTM or Time Series Model is the best fuse model as the RMSE, MAE and MAPE is least among all models and treated as the best model with a volatility of 0.20. Similarly, from Granger Causality Test it was found that volatility of HCL Technologies granger causes the movement of NIFTY 50 and SENSEX. But Volatility Index has no effect on stock price of HCL Technologies. So, the same stock may be taken into consideration for investment by the Rational Investors.
This empirical study investigates the volatility of HCL Technologies' stock prices using a comprehensive approach that combines traditional econometric models with modern fused methodologies. The analysis spans daily data from April 2013 to September 2024. The ARCH family of models is employed to estimate volatility, while a unit root test confirms the stationarity of the time series data. This study examines the volatility of HCL Technologies stock prices using a hybrid approach that combines classical econometric models with modern fused models. The data is daily from April 2013 to September 2024. ARCH family is used to estimate volatility and unit root test is done to confirm the stationarity of the time series data. Among classical models FIEGARCH (1, 1) is found to be the best model as per AIC and SIC values.
The study also uses a fused model approach where the discrete time series data is denoised using Discrete Fourier Transform (DFT) followed by Inverse Fourier Transform. The denoised data is then used as input for Long Short-Term Memory (LSTM) models. The results show that the fused model performs better than classical models with lowest RMSE, MAE, MAPE and volatility of 0.20. Granger Causality Test shows that HCL Technologies volatility affects NIFTY 50 and SENSEX but Volatility Index has no effect on its stock price. So HCL Technologies stock is a good option for rational investors. All the models in this study are significant.

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