Impact of Coiflet Wavelet Decomposition on Forecasting Accuracy: Shifts in ARIMA and Exponential Smoothing Performance

Mohit Kumar, Jatinder Kumar

Department of Mathematics, Guru Nanak Dev University, India.
Email: jatinder.math@gndu.ac.in

Abstract: Accurate electricity demand forecasting is essential for efficient energy management and resource allocation. This study investigates the impact of Coiflet wavelet decomposition on the forecasting performance of Exponential Smoothing (ES) and ARIMA models. Two experimental approaches were employed: one using raw data and another incorporating wavelet denoising. Without wavelet transformation, ES performed better than ARIMA in the testing phase, with RMSE values of 13.62 and 13.93, MAE values of 11.22 and 11.54, and MAPE values of 3.04% and 3.14%, respectively. However, after applying wavelet decomposition, ARIMA showed significant improvement, reducing RMSE by 24.6%, MAE by 23.7%, and MAPE by 23.5% in the testing phase, outperforming ES. The hybrid ARIMA-wavelet model emerged as the most robust approach for forecasting electricity demand, demonstrating the effectiveness of wavelet-based denoising in improving predictive accuracy. These findings highlight the potential of integrating wavelet analysis with statistical forecasting models for more reliable time series predictions.

Keywords: Time series analysis, exponential smoothing, ARIMA, wavelet analysis, KPI.

1. Introduction

Electricity demand in a nation is predominantly influenced by factors such as demographic growth, economic activity, technological innovation, and evolving consumer preferences [18]. In the United States, this demand has been molded by a complex interplay of these elements, resulting in a dynamic and ever-changing energy landscape [19]. As the population expands and economic development progresses, the need for electricity to accommodate the growing requirements of households, businesses, and industries continues to rise [20]. Recent years have seen a pronounced focus on advancing energy efficiency initiatives [18]. It is crucial to recognize that the electricity demand scenario is perpetually evolving, affected by shifts in technology, regulatory frameworks, market conditions, and unpredictable events [18].

Precise forecasting of electricity demand is essential for utility companies and grid operators to manage resources effectively. Advanced notice of expected demand enables these entities to ensure adequate capacity for power generation, transmission, and distribution, thereby preventing shortages and avoiding overloads [13]. Accurate forecasts allow energy providers to optimize their generation schedules and energy mix, leading to reduced operational costs and a lower environmental impact. Unforeseen demand spikes or drops can disrupt power supply, potentially causing blackouts or grid failures [50]. Reliable forecasts are critical for implementing load balancing strategies, ensuring a stable and uninterrupted power supply. Furthermore, electricity demand forecasts are invaluable for policymakers and researchers

aiming to understand consumption trends, assess the impacts of climate change on energy use, and shape energy policies and environmental regulations accordingly [12].
To accurately forecast future values, time series analysis utilizes various modeling techniques to explore the historical relationships between variables. Among the widely adopted statistical models are the Box-Jenkins ARIMA (Autoregressive Integrated Moving Average) and Exponential Smoothing (ES) models, which are capable of addressing a broad spectrum of patterns, including stationary, non-stationary, and seasonal time series [30,32,52]. However, these models often struggle in non-linear scenarios, where the data does not follow a linear trend over time [2,4,24,30]. In such cases, wavelet analysis proves advantageous by detecting high-frequency components in time series data, which enhances the forecasting of non-linear patterns [9,10,14,28,31,33,39]. By employing discrete wavelets, time series of varying lengths can be decomposed into separate component series, each of which can be managed individually for forecasting purposes [25,35,37,40,45,47,49,53]. Wavelets offer greater flexibility and precision in predictions compared to traditional methods [11,16,23,54,62].
To forecast electricity demand in the United States, this study concentrates on creating hybrid models that accommodate the dynamic characteristics of the time series data. Hybrid modeling has proven to be an effective approach for handling such complex time series [29,53,63]. The analysis utilized the USA electricity demand dataset, selecting "demand" as the primary variable of interest from a range of other variables, including coal, gas, hydro, clean energy, bio-energy, CO2 intensity, fossil fuels, and solar energy [Data Source: https://ember-climate.org/data-catalogue/monthly-electricity-data/].

2. METHODOLOGY

In this paper, we utilized Sigma XL and MATLAB software for our research. We analyzed monthly time series data on U.S. electricity demand from https://ember-climate.org/data-catalogue/monthly-electricity-data/, spanning from January 1, 2001, to March 1, 2023, comprising a total of 267 observations. The dataset was split into training and testing phases [26]. During the training phase, we developed predictive models for each decomposed component of the original time series. The following steps were employed in the proposed methodology (see Figure 1):

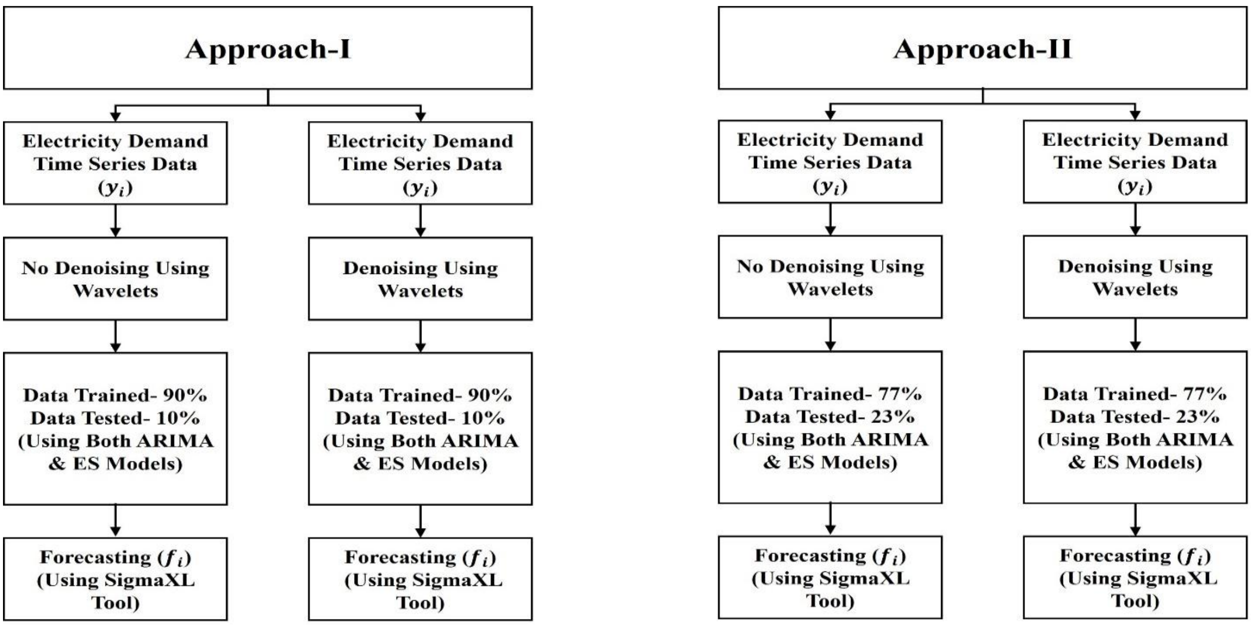


Figure 1: Proposed Technique

Two approaches were employed in our study: First, we utilized 243 of the 267 total readings for training, reserving the remaining 24 readings for testing, which corresponds to a 90% training and 10% testing split. Second, we used 207 readings for training and the remaining 60 readings for testing, resulting in a 77% training and 23% testing split. These two approaches were designed to assess how variations in the training and testing phases affect the model's performance.

2.1 Wavelet Analysis

Wavelets are compact support localized functions with zero mean that can analyze transient and non-periodic signals [8,36,41,42].

A function $\Psi(x) \in L^2(R)$ is called wavelet if it satisfies the following properties:

1) $$\int_{-\infty}^{\infty} \Psi(x)dx = 0 \qquad \ldots (1)$$

2) $$C_\Psi = \int_{-\infty}^{\infty} \frac{|\hat{\Psi}(\omega)|^2}{|\omega|} < \infty \qquad \ldots (2)$$

where $\hat{\Psi}(\omega)$ denotes Fourier transform of $\Psi(\omega)$. Basically, wavelets are a family of functions constructed by dilations and translations of a single function $\Psi(x) \in L^2(R)$ known as 'Mother wavelet' (where the scaling function is called 'Father wavelet'). The family of wavelets $\Psi_{a,b}(x)$ are defined as:

$$\Psi_{a,b}(x) = |a|^{-1/2}\, \Psi\left(\frac{x-b}{a}\right)\ ; a, b \in R,\ a \neq 0 \qquad \ldots (3)$$

where 'a' is a scaling parameter and 'b' is translation parameter.

For discrete wavelet decomposition of time series $\{f(t): t = 1, 2, 3, \ldots\}$, the mother wavelet function $\Psi_{j,k}$ and the father wavelet function $\varphi_{J,K}$ are defined respectively

$$\Psi_{j,k}(x) = 2^{-j/2}\, \Psi(2^{-j}x - k) \qquad \ldots(4)$$

$$\varphi_{J,k}(x) = 2^{-J/2}\, \Psi(2^{-J}x - k) \qquad \ldots(5)$$

The approximation coefficients $\alpha_{J,K}$ are obtained by convoluting the scaling coefficients $\varphi_{J,K}$ with $f(t)$ and convolution with $f(t)$ of the wavelet function $\Psi_{j,k}$ gives the detailed coefficients which are given as below

$$\alpha_{J,k} = \int_{-\infty}^{\infty} f(t)\varphi_{J,k}dt \qquad \ldots (6)$$

$$\beta_{j,k} = \int_{-\infty}^{\infty} f(t)\Psi_{j,k}dt \qquad \ldots (7)$$

Using above integrals, decomposed series applicable to continuous time series $f(t)$ is given by

$$f(t) = \sum_{k\in Z} \alpha_{J,k}\, \varphi_{J,k}(t) + \sum_{j=1}^{J} \sum_{k\in Z} \beta_{j,k}\, \Psi_{j,k}(t) \qquad \ldots (8)$$

Since the time series data under study is discrete and is of finite length, so the discretized time series $y(t)$ of length $K=2^j$ is given by

$$f(t) = \sum_{k=-\infty}^{2^{J-k}-1} \alpha_{J,k}\, \varphi_{J,k}(t) + \sum_{j=1}^{J} \sum_{k=-\infty}^{2^{J-k}-1} \beta_{j,k}\, \Psi_{j,k}(t) \qquad \ldots (9)$$

The decomposition of $f(t)$ into approximation and detail components is also classified in Figure 2 [48].

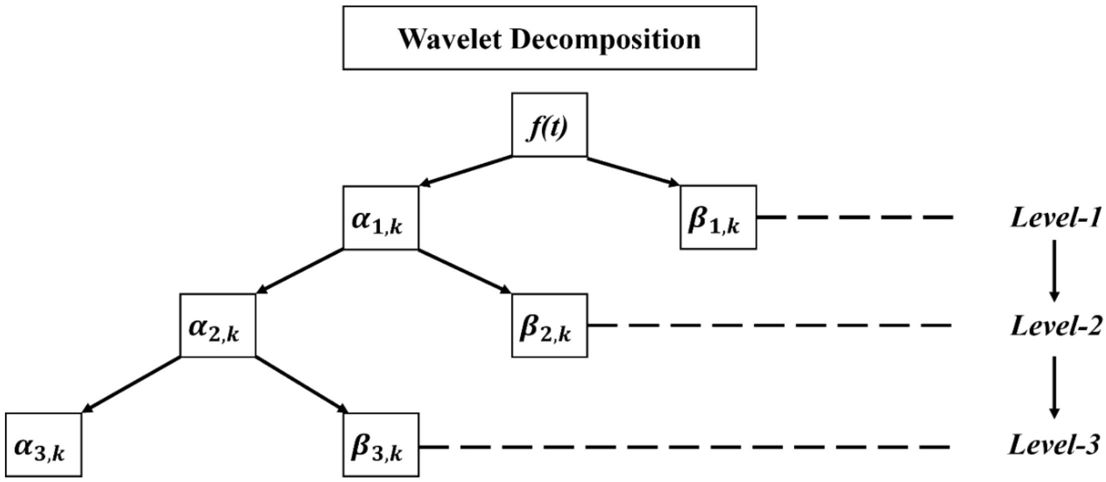


Figure 2

2.2 Forecasting Time Series Models

2.2.1 ARIMA Model

The ARIMA model is the best econometric model available; it outperforms the ARMA, MA, and AR models (Autoregressive moving averages, moving averages, and autoregressive respectively). The ARIMA model takes its cues from the 1960 Box-Jenkins Model, which predicts future time series values based on past data. Parameter estimation, model diagnostic checking, and model identification are the three primary phases of the ARIMA modeling approach. Model identification is the step that comes before parameter estimation, when the time series for stationarity and seasonality are modeled. Time series stationarity can be evaluated using an autocorrelation function (ACF) plot. Application of a differencing transformation can yield stationary data in the event that the time series is non-stationary. The autocorrelation function (ACF) and partial autocorrelation function (PACF) plots can be generated using seasonal differencing to model seasonality. Finding the values of the parameters p and q is another benefit of these plots [5,7]. By using maximum likelihood, a widely used evaluation technique, one can estimate the parameters of the suitably chosen model. The model's overall adequacy is lastly confirmed using the Ljung and Box test to make sure that no additional time series modeling is required [38].

An ARIMA (p, d, q) model using lag polynomial L is expressed

$$\left(1-\sum_{i=1}^{p}\varphi_i L^i\right)(1-L)^d=\left(1+\sum_{j=1}^{q}\theta_j L^j\right)\varepsilon_j \qquad \dots (10)$$

where the non-negative integers p and q are the orders of autoregressive and moving average polynomials respectively; d is the non-seasonal differencing required to make data stationary; f (t) is the value of observations and ε t is a random error at time t; φ_i and θ_j are the coefficients.

2.2.2 Exponential Smoothing Model

The simple exponential smoothing (SES) was initially introduced by Muth, who demonstrated that SES offers optimal forecasts for a random walk, with added noise [22]. Afterwards, Pagels divided trends and seasonal patterns into two categories: multiplicative/nonlinear and additive/linear [56]. In general, exponential smoothing techniques are thought of as a collection of techniques for forecasting certain kinds of univariate time series data [56]. Box and Jenkins, Roberts, Abraham, and Ledolter demonstrated that some linear exponential smoothing techniques can be thought of as special cases of ARIMA models, which advanced the development of a statistical framework for exponential smoothing [44,61]. This approach to time series forecasting is basically used when the data show neither a trend nor a seasonal pattern [57,56]. The equation of simple exponential smoothing is given by:

$$S_t=\alpha X_t+(1-\alpha)S_{t-1}=S_{t-1}+\alpha(X_t-S_{t-1}) \qquad \dots (11)$$

Additionally, the double (Holt's trend corrected method) exponential smoothing model is used when the data exhibits a linear trend and no seasonal pattern [57,27,56]. Adding a term to account for the possibility that a series will exhibit a trend is the main idea behind double exponential smoothing [57,27,56]. The equations are given by:

$S_1 = X_1$, $b_1 = X_1 - X_0$, For $t > 1$,

$$S_t = X_t + (1 - \alpha)(S_{t-1} + b_{t-1}) \quad \dots (12)$$

$$\beta_t = \beta(S_t - S_{t-1}) + (1 - \beta)b_{t-1} \quad \dots (13)$$

Here, S_t = smoothed statistic (simple weighted average of current observation X_t), S_{t-1}= previous smoothed statistic, α = smoothing factor of data; $0 < \alpha < 1$, t = time period, b_t = best estimate of trend at time t and β = trend smoothing factor; $0 < \beta < 1$ [57,27,56].

The triple (Holt-Winter's exponential) exponential smoothing model is used when the data exhibits both a linear trend and a seasonal pattern [44,21,61,57]. This approach has been employed by us for our research purpose. The equations are given by

$$S_0 = X_0 \quad \dots (14)$$

$$S_t = \alpha \frac{X_t}{c_{t-1}} + (1 - \alpha)(S_{t-1} + b_{t-1}) \quad \dots (15)$$

$$b_t = \beta(S_t - S_{t-1}) + (1 - \beta)b_{t-1} \quad \dots (16)$$

$$c_t = \gamma \frac{X_t}{S_t} + (1 - \gamma)\, c_{t-1} \quad \dots (17)$$

where, c_t = sequence of seasonal correction factor at time t and γ = seasonal change smoothing factor; $0 < \gamma < 1$ [21,61,57].

2.2.3 Hybrid Time Series Prediction Model

Since wavelet decomposition techniques have varying propensities to handle linear and non-linear data features, the coupled models put forth in this work comprise ARIMA and ES models' forecasting on time series data that has been refined by wavelet decomposition techniques. Through the modeling of both linear and non-linear data components, these coupled models can enhance forecasting performance [43]. Time-series data f(t) is first decomposed into approximations (A_j) and detail (D_j) coefficients (Section 5.1) in the wavelet decomposition method. These coefficients can be used as independent series for forecasting, and each of these series is then modeled and forecasted using a suitable ARIMA & ES model. The predicted approximations ($\widehat{A_j}$) and detail ($\widehat{D_j}$) coefficients so obtained are summed to obtain forecasted data $\widehat{f(t)}$, expressed as

$$\widehat{f(t)} = \widehat{A_j} + \widehat{D_j}\ ;\ j = 1, 2, 3, ... \quad \dots (18)$$

3. Results and Discussions

3.1 Time Series Analysis

Many time series methods begin with a stationarity check of the data. Autocorrelation function (ACF) and partial autocorrelation function (PACF) plots can be used to assess non-stationarity in time series data, which is indicated by rapid changes. The time series is determined to be non-stationary by a slow decaying ACF plot, which is eliminated by differencing transformation to produce stationary data [7]. After checking stationarity, the next step is to determine the order of the ARIMA model parameter, which can be determined by the ACF plot of differenced time series. Then, an appropriate ARIMA model is fitted to data that generates future values of time series data. So, in both the approaches when wavelets were not used, ARIMA (1,1,1) (Figure 4,Figure 5,Figure 6 and Figure 7) is best fit for our dataset type.

Similarly, our data has both linear trend and seasonal pattern, so we used Holt-Winter's exponential smoothing model with parameter estimates α-level smoothing and γ-seasonal smoothing (Table 1) in both the approaches and models are bases on Akaike information criterion (AIC)

Table 1: Showing the values of parameter estimates in Exponential Smoothing Model (Holt-Winter)

ES- Approach-I	α- Level Smoothing	γ- Seasonal Smoothing	ES- Approach-II	α- Level Smoothing	γ- Seasonal Smoothing
Without Wavelet	0.362	0.0001	Without Wavelet	0.362	0.0001
With Wavelet	0.9999	0.0001	With Wavelet	0.9999	0.0001

3.2 Wavelet Decomposition

The mother wavelet, its level, and the decomposition order are critical factors to consider when applying wavelet decomposition to time series. Although there are many families of wavelets for decomposition, one of the significant wavelet types with unique benefits is the Coiflet wavelet. The Coiflet wavelet is used to break down time series data on electricity demand in the United States. The approximations consist of details that represent high-frequency components and low-frequency parts that show a trend. To obtain predicted components, an appropriate ARIMA and ES model are separately modeled for the approximation A_3 and details D_1, D_2, and D_3. The predicted outputs $\widehat{A_3}$, $\widehat{D_1}$, $\widehat{D_2}$ $\widehat{D_3}$are finally summed to obtain the forecasts of demand given in Eq.

$$\widehat{f(t)} = \widehat{A_3} + \widehat{D_1}+\widehat{D_2}+\widehat{D_3} \qquad \dots (12)$$

where capped (^) symbol is used to denote predicted values.

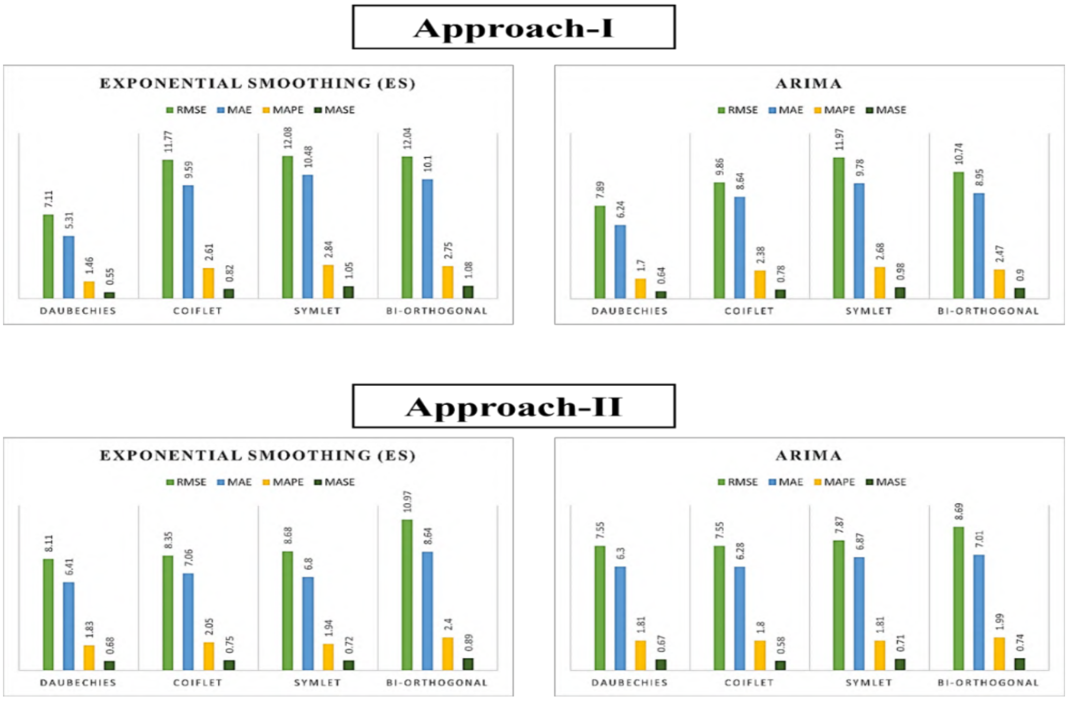


Figure 3: Selection of Best Wavelet for Our Database

So, here we tested approximately all the types of above wavelet available in the MATLAB tool box to denoise the signal then came to the result with full statistical analysis based on Akaike information criterion (AIC) that Coiflet wavelet is best for our research purpose (Figure 3).

3.3 Hybrid Time Series Model

Initially, various wavelet functions, such as Daubechies, Symlet, Coiflet, and Biorthogonal are applied to decompose the USA electricity demand dataset into different frequency components. The decomposed series are then reconstructed using selected wavelet coefficients to obtain denoised data. Subsequently, both ES and ARIMA models are applied to the original and denoised time series for forecasting purposes. The Coiflet wavelet comes out to be the best fit for this kind of dataset for both the approaches.

Autoregressive Integrated Moving Average (ARIMA) and Exponential Smoothing (ES) are models used for generating values independently or, in conjunction, with Wavelet decomposition [1,3,15,44]. In these models the data is first broken down into series using the Coiflet wavelet. Then both ARIMA and ES models are applied to each constituent series to create a forecast. Finally, the predicted values of the constituent series are added together to obtain the output of the model. In approach-I and approach-II, ARIMA (2,1,1) (See Figure 9) and ARIMA (2,1,5) (See Figure 10) are comes out to be the best models for the denoised dataset and are selected on the bases of Akaike information criterion (AIC) with the significance limit alpha = 0.05. The predictive performance hybrid models and ARIMA & ES models are compared finally to find the best model among them with least forecasting errors.

For the Model evaluation we have used four standard error measures named as Key Performing Indicators (KPIs) and these are defined as:

$$\mathrm{RMSE} = \sqrt{\sum_{i=1}^{n} \frac{(y_i - f_i)^2}{n}}$$

$$\mathrm{MAE} = \frac{\sum_{i=1}^{n} |y_i - f_i|}{n}$$

$$\mathrm{MAPE} = \frac{1}{n} \sum_{i=1}^{n} \frac{|y_i - f_i|}{y_i}$$

$$\mathrm{MASE} = \frac{\mathrm{MAE}}{\frac{1}{n-1} \sum_{i=2}^{n} |y_i - y_{i-1}|}$$

Where y_i's are the actual values, f_i's are the forecasted values and n is the total number of observations.

3.3.1 Results and Discussion – Without Wavelet

Table 2: ES on Original Time Series Data without using wavelet with Approach-I

KPIs	Training Phase (Values)	Testing Phase (Values)
RMSE	8.58	13.62
MAE	6.82	11.22
MAPE	1.99	3.04
MASE	0.67	1.11

Table 3: ARIMA on Original Time Series Data without using wavelet with Approach-I

KPIs	Training Phase (Values)	Testing Phase (Values)
RMSE	9.26	13.93
MAE	7.42	11.54
MAPE	2.17	3.14
MASE	0.73	1.14

Table 4: ES on Original Time Series Data without using wavelet in Approach-II

KPIs	Training Phase (Values)	Testing Phase (Values)
RMSE	8.50	10.06
MAE	6.75	8.07
MAPE	1.98	2.40
MASE	0.66	0.860

Table 5: ARIMA on Original Time Series Data without using wavelet in Approach-II

KPIs	Training Phase (Values)	Testing Phase (Values)
RMSE	9.43	10.14
MAE	7.58	8.35
MAPE	2.22	2.38

MASE	0.74	0.85

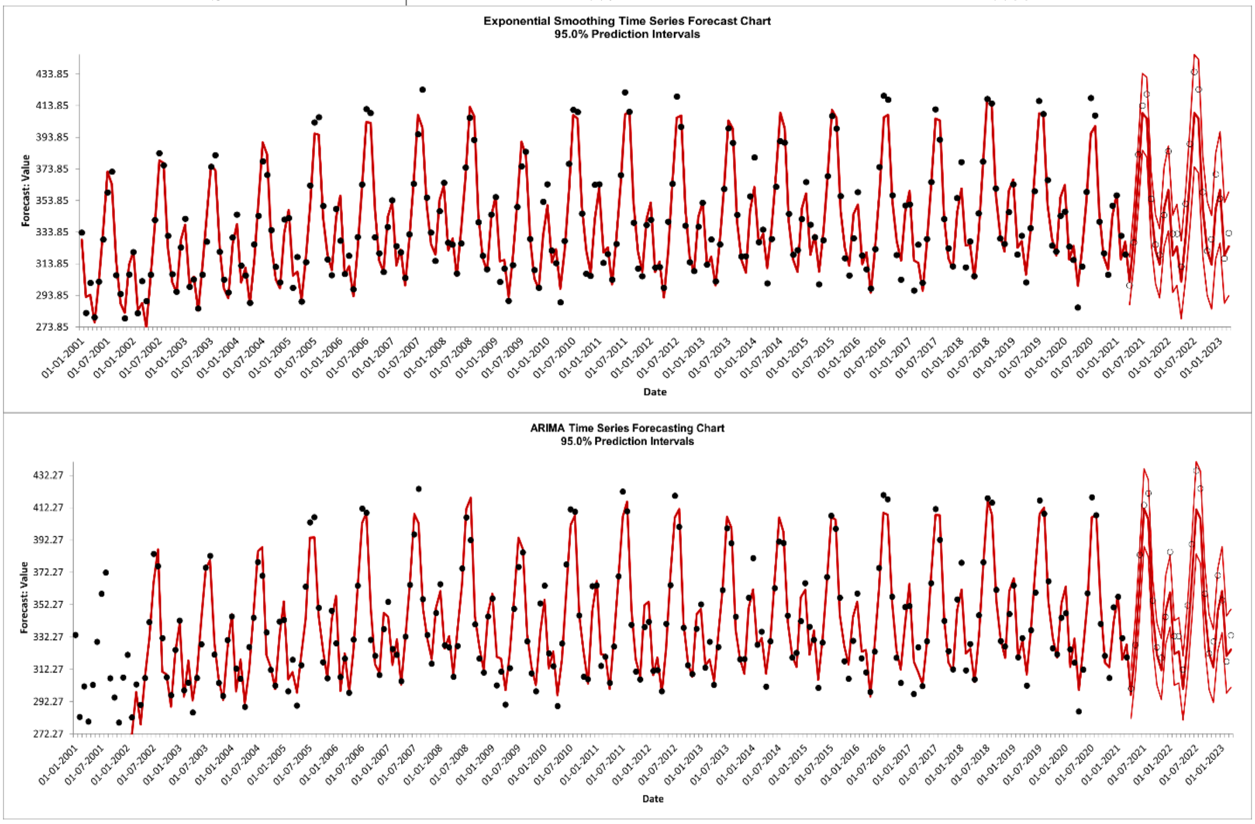
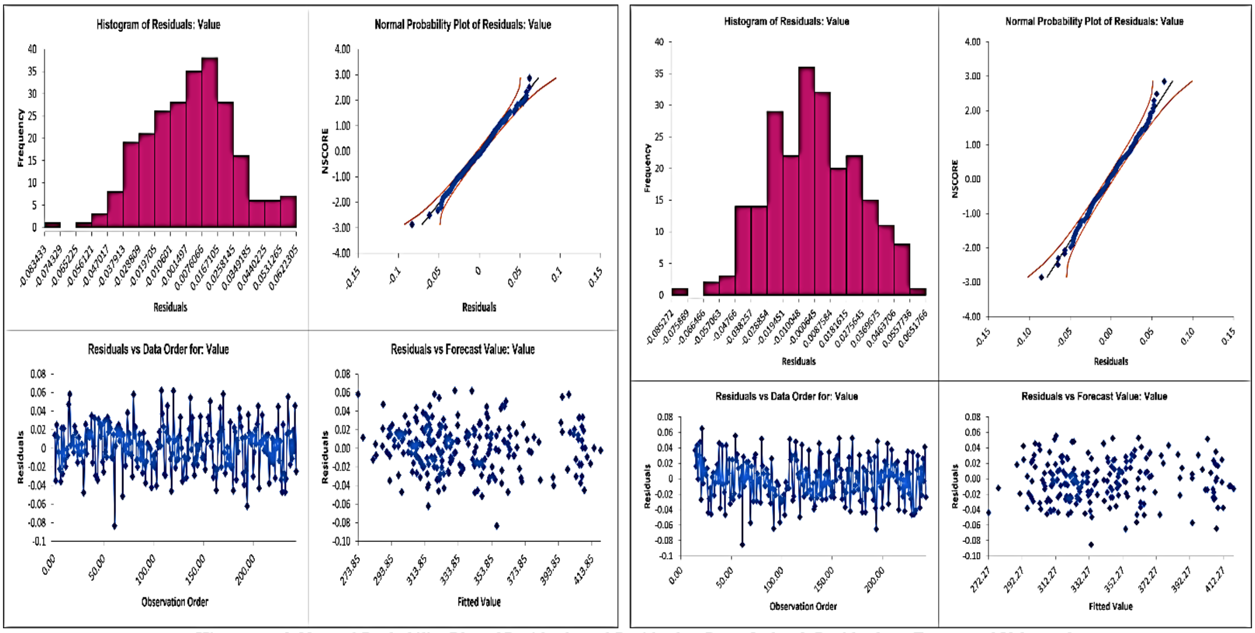


Figure 4: Performance of ES (Holt-Winter Method) and ARIMA (1,1,1) without wavelet in Approach-I



Histogram & Normal Probability Plot of Residuals and Residual vs Data Order & Residuals vs Forecasted Values of ES Model (Left) and ARIMA Model (Right)- Approach-I

Figure 5

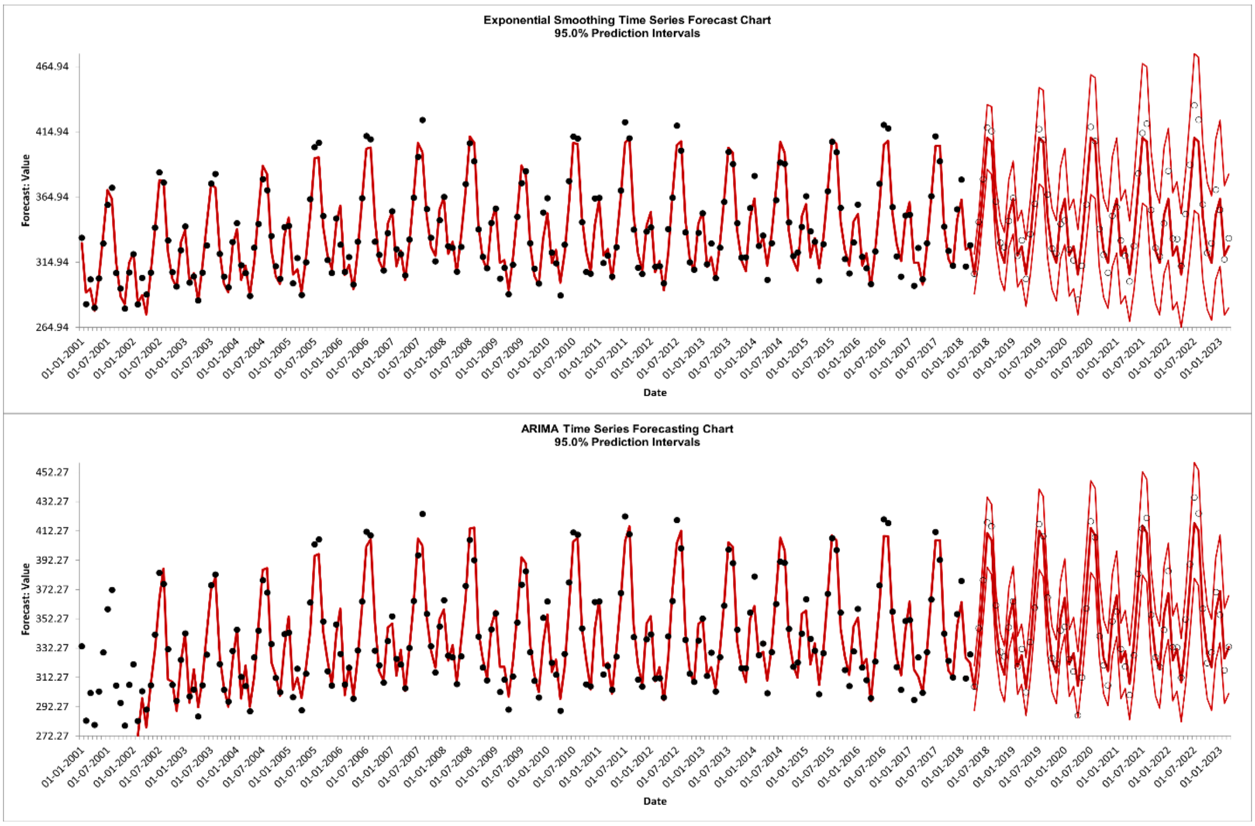
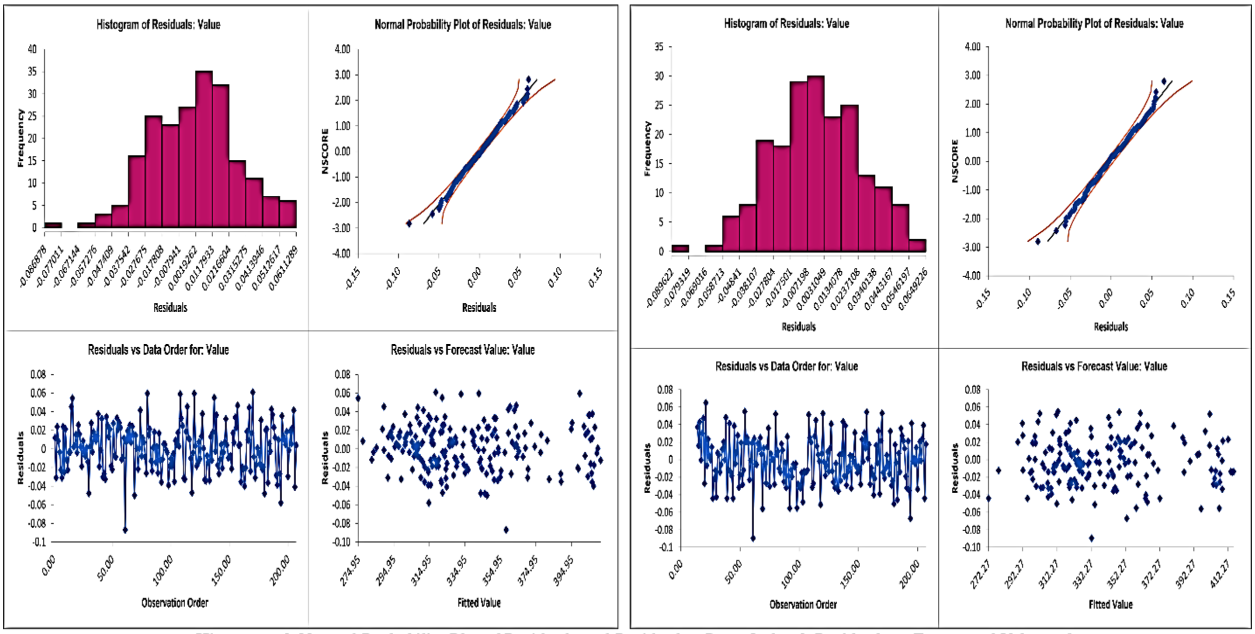
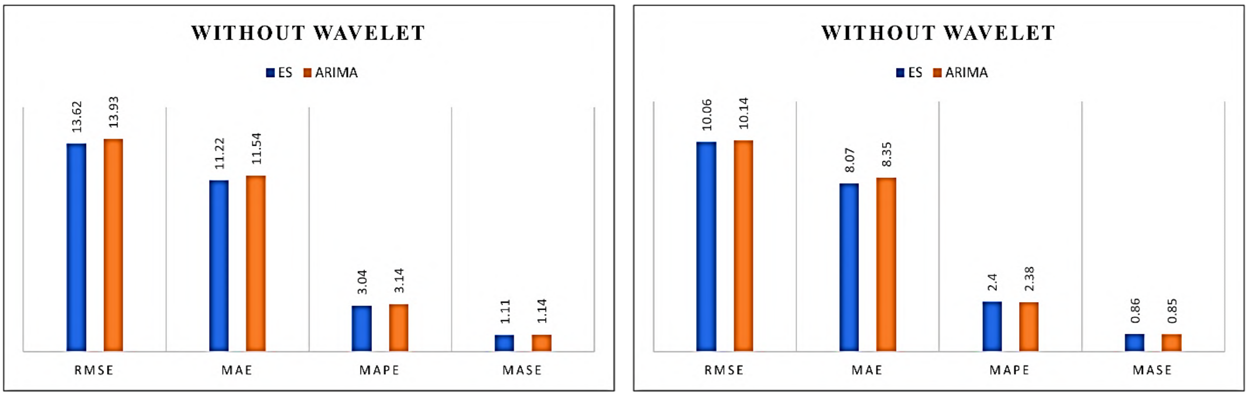


Figure 6: Performance of ES (Holt-Winter Method) and ARIMA (1,1,1) without wavelet in Approach-II



Histogram & Normal Probability Plot of Residuals and Residual vs Data Order & Residuals vs Forecasted Values of ES Model (Left) and ARIMA Model (Right) – Approach-II

Figure 7



Comparison of ES and ARIMA Model without using Wavelet in Approach-I (Left) and Approach-II (Right)

Figure 8

The tables (Table 2, Table 3, Table 4, and Table 5) above, as well as Figure 8, make it abundantly evident that the Exponential Smoothing (ES) model outperforms the ARIMA model in nearly every KPI measure. However, in this case, the data is not denoised using wavelet. Let's now examine the results by applying the wavelet to the original time series data.

6.3.2 Result and Discussion – With Wavelet

Table 6: ES on Denoised Time Series Data using wavelet with Approach-I

KPIs	Training Phase (Values)	Testing Phase (Values)
RMSE	3.25	11.77
MAE	2.50	9.59
MAPE	0.73	2.61
MASE	0.28	0.82

Table 7: ARIMA on Denoised Time Series Data using wavelet with Approach-I

KPIs	Training Phase (Values)	Testing Phase (Values)
RMSE	2.74	9.86
MAE	1.96	8.64
MAPE	0.57	2.38
MASE	0.22	0.78

Table 8: ES on Denoised Time Series Data using wavelet with Approach-II

KPIs	Training Phase (Values)	Testing Phase (Values)
RMSE	3.37	8.35
MAE	2.58	7.06
MAPE	0.76	2.05
MASE	0.29	0.75

Table 9: ARIMA on Denoised Time Series Data using wavelet with Approach-II

KPIs	Training Phase (Values)	Testing Phase (Values)
RMSE	2.96	7.55
MAE	2.16	6.28
MAPE	0.64	1.80
MASE	0.23	0.58

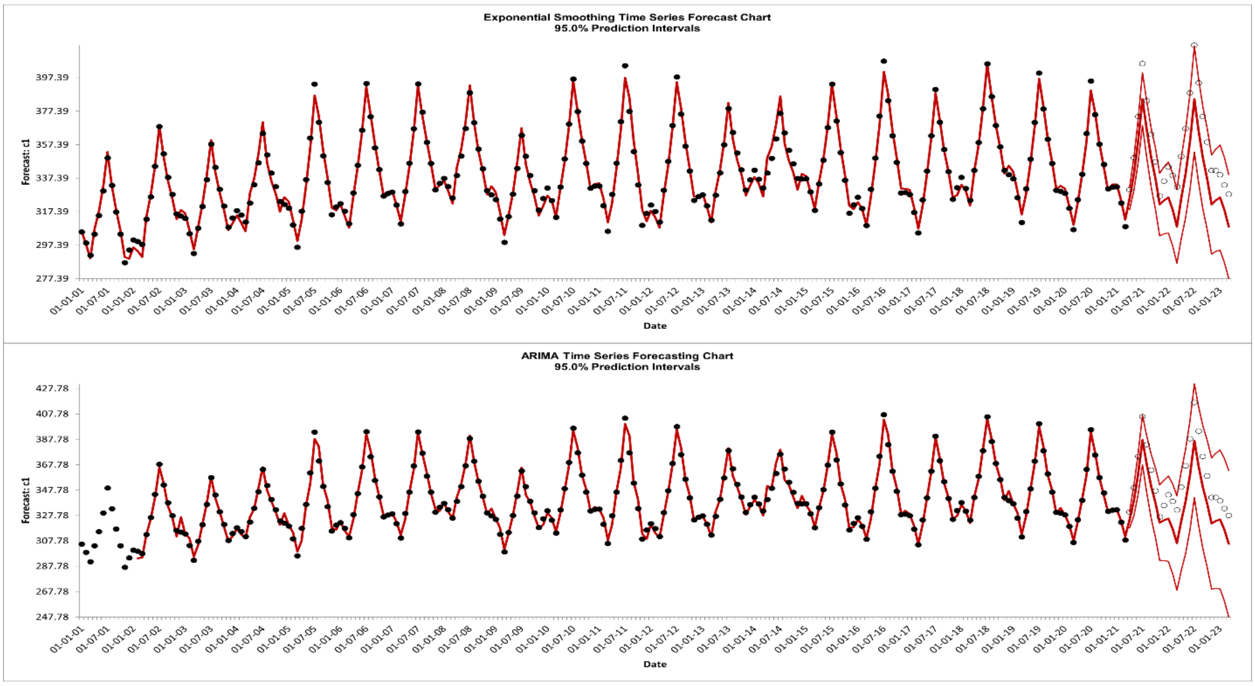


Figure 9: Performance of ES (Holt-Winter Method) and ARIMA (2,1,1) without wavelet in Approach-I

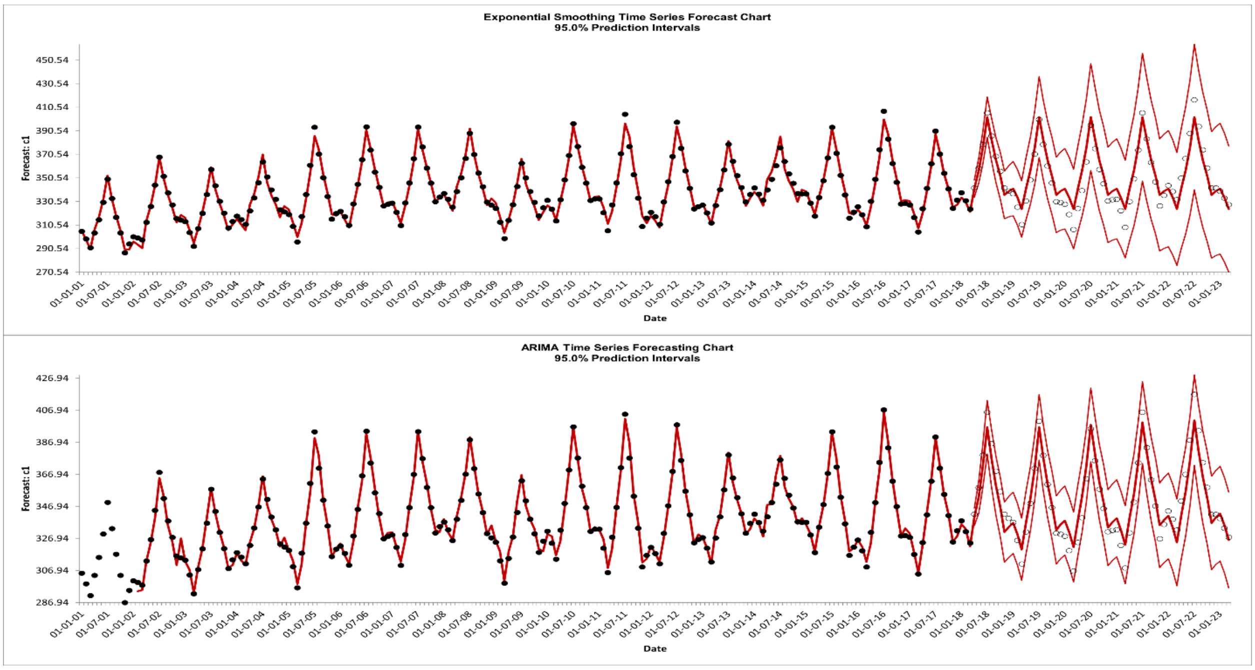
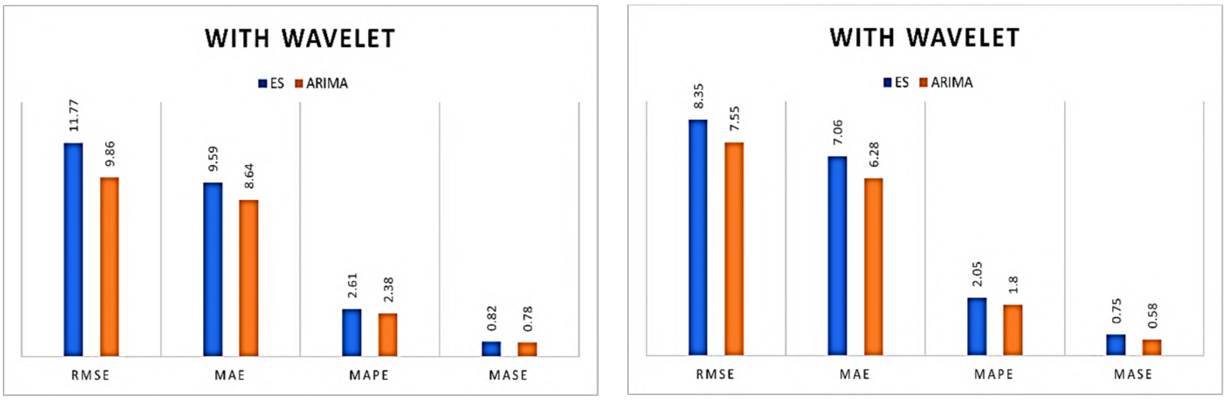


Figure 10: Performance of ES (Holt-Winter Method) and ARIMA (2,1,5) without wavelet in Approach-II



Comparison of ES and ARIMA Model using Wavelet in Approach-I (Left) and Approach-II (Right)

Figure 11

The tables (Table 6, Table 7, Table 8, and Table 9) above, as well as Figure 11, ARIMA model outperforms the ES model in every KPI measure.

3.4 Impact of Coiflet Wavelet Over ES and ARIMA Models

Here, we see that in testing phase ARIMA performing better than ES model when wavelets are applied. This is what we called as impact of wavelet over ES and ARIMA model because if we refer to fig. 8, it is clearcut that ES is better than ARIMA but if we refer to figure 11, ARIMA is showing better results than ES which is the scenario of wavelets.

4. CONCLUSION

This study examined the impact of Coiflet wavelet decomposition on the forecasting accuracy of Exponential Smoothing (ES) and ARIMA models for electricity demand prediction. Initially, when applied to raw data, ES outperformed ARIMA, with lower RMSE (13.62 compared to 13.93), MAE (11.22 compared to 11.54), and MAPE (3.04% compared to 3.14%). However, after applying wavelet decomposition, ARIMA demonstrated significant improvements, with RMSE decreasing by 24.6% (from 13.93 to 9.86), MAE decreasing by 23.7% (from 11.54 to 8.64), and MAPE reducing by 23.5% (from 3.14% to 2.38%). This shift indicates that wavelet preprocessing effectively enhances ARIMA’s predictive capabilities, making it a superior choice for noisy time series data.

The results suggest that while ES performs well in conventional settings, ARIMA benefits more from wavelet-based denoising, leading to enhanced accuracy. Therefore, integrating Coiflet wavelets with ARIMA models can significantly improve forecasting reliability in energy management and other practical applications. Future research could explore different wavelet families and hybrid approaches, incorporating machine learning models to further optimize forecasting performance.

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DATA AVAILABILITY

The monthly USA electricity demand time series data that is used in this study is available for download from the site of Ember Climate with the given link https://ember-climate.org/data-catalogue/monthly-electricity-data/

CONFLICT OF INTEREST

The author declares that there is no conflict of interest.

STATEMENTS AND DECLARATION
All authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript.

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