Free Convective Thermal Transmission through Distinct Geometrical Shapes

Abrar Al-hadad, Qusay Rasheed Al-amir

Mechanical Power Engineering Department, College of Engineering and Technologies, Al-Mustaqbal University, Iraq.
Email: Abrar.Abdulkareem.Saeed@uomus.edu.iq

Abstract: A numerical study was done to analyze the conjugate convective thermal transmission through an enclosure containing a solid object. The geometrical shape of the inserted object was changed many times to check the influence of changing the geometry on both heat transfer process and fluid flow. COMSOL Multiphysics 6.0 was utilized to present the findings in terms of isotherms , stream function and Nusslet number. The working fluid here is air of prandtle number of 0.7. The effect of Ra on both Nu and the heat transfer process was studied here. There is also a comparison between the results of the different geometries to check which one has achieved the best results. It is worthy to mention that a validation process was done with a previous study to check the validity of the past results. The application of this field is so common in nuclear reactors and energy storage.

1. Introduction

Free convective mode of thermal transmission through a heated enclosure has always been a matter of concern due to its significance in lots of applications. Free convection is preferred over forced convection since it is considered not noisy like forced convection. Nuclear reactors, energy storage, and solar collector technologies are the most prominent applications[1]. Another application of this field is constructing energy components such as electronic enclosure. Inserting a solid object of high thermal conductivity inside the enclosure will improve the thermal transmission rate. A. C. Baytas et al.,2001[2] presented a numerical study of conjugate free convection in a square cavity filled with porous medium. The average Nu was calculated for the chosen geometry. It was shown that the interface T dropped with the raise of the wall thermal conductivity k. Fu-Yun Zhao et al.,2006[3] presented a theoretical study of conjugate conduction and free convection in an enclosure using inner as well as otter heat sources. Results were presented in terms of streamlines , isotherms, and Nu within wide range of Ra. It was found that using a large heat source was recommended to maintain zero velocity at the solid area. Maged A.I. El-Shaarawi et al.,2007[4] investigated numerically conjugate free convection in a an annuli. FDM was utilized to solve the bipolar equations. Prandtle number value of the considered fluid was 0.7. It was found that increasing the eccentricity will increase the induced flow rate, also raising the radius ratio increases the total absorbed thermal energy. Fu-Yun Zhao et al.,2007 [5]presented a numerical study of conjugate free convective thermal transmission through an enclosure. The results of fluid flow and heat transfer were presented in terms of streamlines and heatline contours. It was found that a body with a relative low thermal conductivity will improve the heat transfer through the enclosure. Jamal A. et al.,2008[6] presented a numerical study of conjugate free convection in a vertical annulus. A Newtonian fluid of Pr=0.7 was considered as working fluid. FDM was utilized to solve the PDE. Results showed that raising the eccentricity will raise the rate of fluid flow as well as the heat transfer rate. Geniy V. Kuznetsov and Mikhail A. Sheremet 2009[7] investigated numerically the unsteady conjugate natural convection in an enclosure. The 3-D equations were written in the non-dimensional form. An implicit FDM was used to solve the equations. It was found that the average Nu increases as Gr increase. Mohamed A. Antar and Hasan Baig 2009[8] investigated numerically the conjugate thermal transmission in a hollow block. Two dimensional formula equations were used. It was found that when the number of cavities increase will lead to a heat loss decrease while keeping the width of the block constant. P. Venkata Reddy et al.,2010[9] presented a numerical study of conjugate natural convection in a vertical annulus containing a heat generating rod at the center. Air was considered as working fluid. It was found that there was a decrease in the dimensionless temperature when increasing Grashof number. Madhusudhana Gavara and P. Rajesh Kanna 2012[10] presented a numerical study of free convective thermal transmission in a rectangular cylinder. Non-dimensional temperature and Nu was analyzed. It was found that the heat transfer rates at the bottom of the cylinder was higher than the other locations. Consequently Nu was found to be 2.5 times higher than Nu at the other locations. N. Bianco et. Al., 2012[11] presented a numerical analyzation using COMSOL application to study convective-conductive-radiative thermal transmission through the considered domain. The velocity distribution, pressure, temperature, and fluid flow were studied. It was found that higher temperatures were attained in the solid phase than in the fluid phase. Didier Gossard and Berangere Lartigue2013[12] presented a three dimensional numerical study of conductive-convective-radiative thermal transmission through air filled cavities. The cavities were closed at bottom and top. An experimental model was built to simulate the theoretical one. The measured temperatures were compared to the theoretical ones and were found to be very close. It was found that a good agreement was attained by comparing the experimental readings of temperatures with the theoretical ones. Tanmay Basak et. Al.,2013[13] analyzed numerically thermal transmission conjugate convective mode through a square cavity. Two thicknesses of the wall were considered in the study. The results were showed in terms of isothermal lines and streamlines. It was found that heat transfer rates were maintained constant at low values of thermal conductivity ratio. The average Nu was found to be high at larger values of thermal conductivity ratio K. Abidi Saad aissa et. Al.,2014[14] analyzed numerically conjugate free convective mode of thermal transmission through a triangular geometry inserted in a square cavity. The considered fluid was air of Pr = 0.71. A wide range of Ra values was taken in this study. It was noticed that the maximum attained value of average Nu reached to 15.555 which is considered a good rate of heat transfer. Vineeth V. K et.al.,2016[15] presented theoretical two dimensional study of free convective mode of thermal transmission in a square enclosure heated from the bottom. FLUENT 14.0. was used to do the numerical study. Air and other nanofluid were considered in this study. Values of Rayliegh number ranged from 5x10^7 to 25x10^7. Results revealed that the rate of heat transfer increases with raising the hot wall temperature. Khaled Al-Farhany and Ammar Abdulkadhim,2018[16] presented a theoretical study of conjugate free convective thermal transmission in a partially warmed enclosure using COMSOL Multiphysics. A non- dimensional form of the PDE were used with different values of Darcy number and modified Ra were considered. Results revealed that increasing the length of the thermal source will lead to an increase in the local Nu. It was also noticed that at higher values of Ra the local Nu for solid and fluid will raise, consequently the heat transfer rate was improved. K . Al-Farhany and Ammar Abdulkadhim,2018[17] analyzed numerically free convective heat transfer in a rectangular wall using FEM. A non-dimensional form of the governing equations was used to solve the problem by applying the boundary conditions. Results revealed that Nu decreased as the aspect ratio raised. It also revealed that Nu raised as the modified Ra raises. Sumon Saha et. Al.,2020[18] analyzed numerically 2-D conjugate laminar free convective thermal transmission through differentially heated square cavity. All the governing PDE were solved using FEM. The working fluid in this study was chosen to be air. The considered values of Ra in this study is (10^3 < Ra< 10^9) and Pr of 0.7. It was noticed that the rate of heat transfer increases as increasing the value of od Ra. Ahmad Jamal and Esmail M. A. Mokheimer., 2022[19] investigated numerically laminar natural convection in a vertical annular channel. FDM was employed to solve the governing PDE. Air was chosen to be working fluid with Pr of 0.7. Results revealed that eccentricity and radius ratio raised as thermal conductivity ratio and thickness of the cylinder walls were increased.

2. Mathematical Formulation

The geometry being studied is illustrated in the figure below represented by the square pipe containing a solid object , the shape of the inserted object varies according to the cases being studied. The purpose of using different geometries is to compare the results and check the effect of changing the geometrical shape on the resulting streamlines and isotherms. The mathematical representation of the physical domain is presented in this section. The schematic figure represents a two dimensional model which contains a compound conductive- free convective thermal transmission process through an enclosure containing a solid object inside. The considered working fluid in this study is air (Pr=0.71). Different geometrical shapes were investigated in this paper.

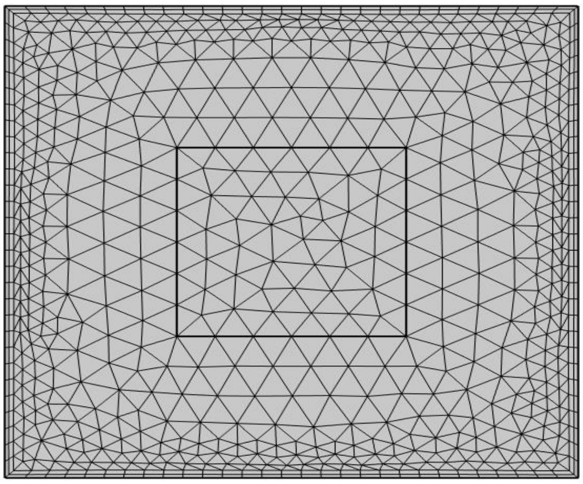


FIGURE1.(a) representation of the meshed physical domain

Representation of the Mathematical Equations

The following equations is the mathematical representation of the physical system that is being studied. Where they describe the conductive- convective thermal transmission process through the square enclosure containing a solid object. The solid object was positioned exactly at the center of the outer geometry. The chosen coordinates system is (x-y) in two dimensions since it was assumed that the dimension in the z-direction was large enough to be neglected. The process of thermal transmission and fluid flow through this geometry is being tested and described using these mathematical formulations. The space between the enclosure and the solid object is filled with the working fluid (air) of pr=0.7. A heat flux of 50 w/m^2 is applied on the top wall of the enclosure while the bottom wall is subjected to a temperature of 300 K. The right and left walls are insulated, also the inner object is insulated from all sides. The radiation is neglected hence the equations of conservation of mass, momentum and energy are stated[20] to describe the steady laminar free convective thermal transmission process.

$$u\ \frac{\partial u}{\partial x} + v\ \frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$
$$u\ \frac{\partial v}{\partial x} + v\ \frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \beta g\,(Tf - Tc)$$
$$u\ \frac{\partial T}{\partial x} + v\ \frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{1}$$

Where, Tf represents the fluids temperature while Tc represents cold temperature. All solid-fluid interfaces are assumed to be no-slip surfaces. Using the non-dimensional variables demonstrated below:

$$X = \frac{x}{l},\quad Y = \frac{y}{l},\quad U = \frac{ul}{\alpha},\ V = \frac{vl}{\alpha}$$
$$\Theta f = \frac{Tf - Tc}{Th - Tc},\quad P = \frac{pl^2}{\alpha\rho},\ Pr = \frac{v}{\alpha},$$
$$Ra = \frac{g\beta\,(Th - Tc)\,\ell 3\ Pr}{v^2},\ \ \Theta s = \frac{Ts - Tc}{Th - Tc}. \tag{2}$$

Hence, the non-dimensional form of the mathematical formulations becomes:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0$$
$$U\frac{\partial U}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right),$$
$$\mathbf{U}\frac{\partial \mathbf{V}}{\partial \mathbf{X}} + \mathbf{V}\frac{\partial \mathbf{V}}{\partial \mathbf{Y}} = -\frac{\partial \mathbf{P}}{\partial \mathbf{X}} + \mathbf{Pr}\left(\frac{\partial^2 \mathbf{V}}{\partial \mathbf{X}^2} + \frac{\partial^2 \mathbf{V}}{\partial \mathbf{Y}^2}\right) + \mathbf{Ra\,Pr\,\Theta f}, \tag{3}$$
$$\mathbf{U}\frac{\partial \mathbf{\Theta f}}{\partial \mathbf{X}} + \mathbf{V}\frac{\partial \mathbf{\Theta f}}{\partial \mathbf{Y}} = \left(\frac{\partial^2 \mathbf{\Theta f}}{\partial \mathbf{X}^2} + \frac{\partial^2 \mathbf{\Theta f}}{\partial \mathbf{Y}^2}\right), \tag{4}$$
$$\frac{\partial^2 \Theta s}{\partial X^2} + \frac{\partial^2 \Theta s}{\partial Y^2} = 0. \tag{5}$$

The dimensionless velocity is assumed to be zero in the solid field and on the solid-fluid interfaces.

Boundary Conditions and Simulation Cases

The boundary conditions of the non-dimensional temperature are specified as following:

$$\Theta f = 1 \ \text{at}\ X = 0,$$
$$\Theta f = 0 \ \text{at}\ X = 1$$
$$\frac{\partial \Theta f}{\partial Y} = 0 \text{ at } Y = 0, Y = 1, \tag{6}$$
$$\mathbf{\Theta f} = \mathbf{\Theta s} \ \text{at the outter solid surface}$$
$$\frac{\partial \Theta f}{\partial \eta} = \mathbf{Kr}\frac{\partial \Theta s}{\partial \eta} \ \text{at the inner solid wall}$$

Where Kr = ks/kf is the thermal conductivity ratio.

- CaseI. Square enclosure of 10 cm side length contains a square solid object of 4 cm side length .
- Case II. Square enclosure of 10 cm side length contains a square solid object of 5 cm side length.
- Case III Square enclosure of 10 cm side length contains an elliptic solid object of 3 cm a- semiaxis and 1cm b-semiaxis .
- Case IV square enclosure of 10 cm side length involves an elliptic solid object of 4 a- semiaxis and 1.5 cm b-semiaxis .

The range of Rayleigh number, $10^3 \leq Ra \leq 10^7$ for each case.

The fluid movement is demonstrated by the stream function Ψ which is computed from velocity components U and V. The relationships are clarified below[20]:

$$\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial Y^2} = \frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X} \qquad (7)$$

The heat transfer rate is given by [20]:

$$\mathbf{Q} = -\mathbf{Kf}\,\frac{\partial \mathbf{Tf}}{\partial \mathbf{X}} \qquad (8)$$

Then it becomes in terms of the dimensionless form:

$$\mathbf{Nu\ loc} = \frac{\mathbf{ql}}{(\mathbf{Th}-\mathbf{Tc})} = -\mathbf{Kf}\,\frac{\partial \mathbf{Tf}}{\partial \mathbf{X}} \qquad (9)$$

where Nu_{loc} is local Nusselt number .The average Nu on the heated surface can be calculated using integration according to the following formula[20] :

$$\mathbf{Nuavg} = \int \quad -\frac{\partial \boldsymbol{\Theta}\mathbf{f}}{\partial \mathbf{X}}\,\mathbf{dY}. \qquad (10)$$

Validation Process

To check the validity of the previous published related research a validation process was done to validate the computed heat lines, the previously published study of conjugate free convective thermal transmission through an enclosure containing a solid object was solved here.

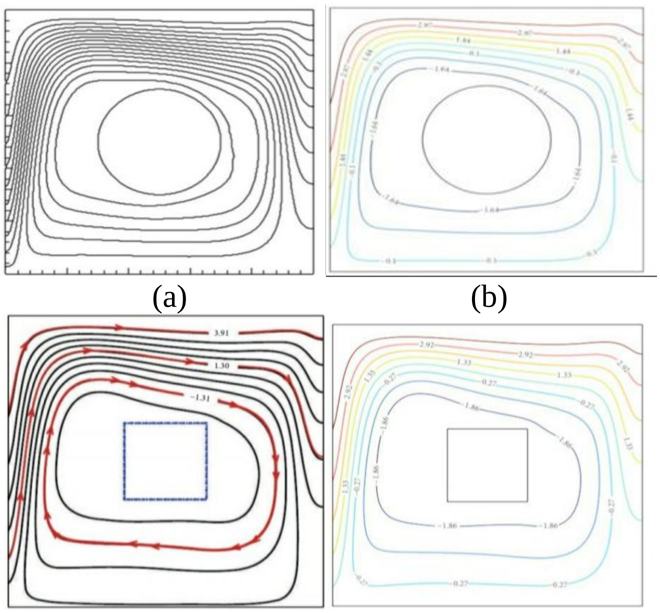


FIGURE2. Comparison of the past heat lines(a) Costa and Raimundo [21] for cylinder,N=∞, (c) of Zhao et al. [5] for quadrilateral N=4 with the present calculated heat lines (b,d) .

Figure2 presents a Comparison of the past calculated heat lines (a,c) against that of Costa and Raimundo [21] for cylinder,N=∞, and (c) of Zhao et al. [5] for quadrilateral N=4. The validation process shows a very good agreement.

3. Computational Methodology

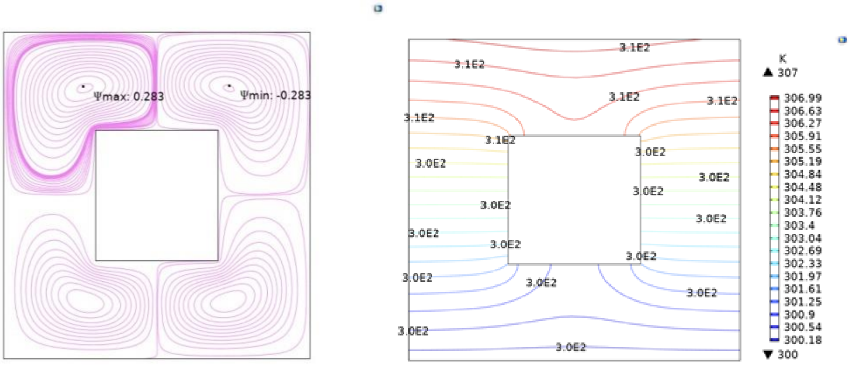
COMSOL Multiphysics 6.0 is the utilized software to solve the governing PDE. Navier-Stokes equations , continuity equation and momentum equation are solved numerically using COMSOL. The model builder in this app. Is used to build, design , simulate and apply physics to the geometry [22]. COMSOL is a feigning program which supplys single-physics and fully coupled multiphysics modeling competences . The convergence criterion is set to 10^{-6}. In the present work, triangles are the elements of the applied mesh.

4. RESULTS AND DISCUSSION

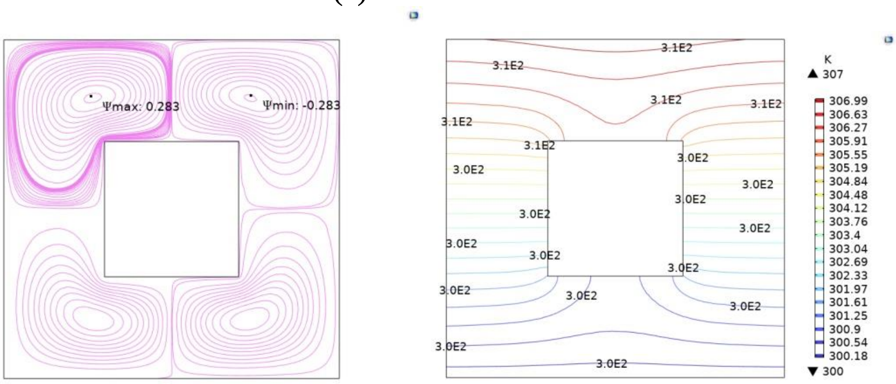
The effect of the geometrical shape on the free convective thermal transmission process through a square enclosure containing a solid object is studied in this paper. Different geometrical shapes of the solid object were used to check the influence on the results. The flow system of the three cases mentioned previously will be demonstrated and discussed. The values of the considered Rayliegh number ranges from 10^3 to 10^7. In addition to , the ratio of the convective to conductive heat transfer will be computed and illustrated.

Figure3 shows the flow system for the first case which consists of a square enclosure containing a square solid object . The contours of streamlines are demonstrated on the left while the isothermal lines are demonstrated on the right. The maximum as well as the minimum values of the stream function is shown on the figure. For figure (3,a) the calculation was made at Ra value of 10^3 . It is observed that there are four groups of swirls around the solid object forming the streams of the free convective flow at four corners around the solid object. The range of the temperatures of the isothermal lines is illustrated on the side of the figure. It is obvious that the highest temperatures are at the top due to the density differences where the convective currents occurs due to this principle.

Figure (3,b) shows the computed results at Ra value of 10^7. The intensity of the swirls increases at higher values of Ra even though the change is not so large but it can be observed.

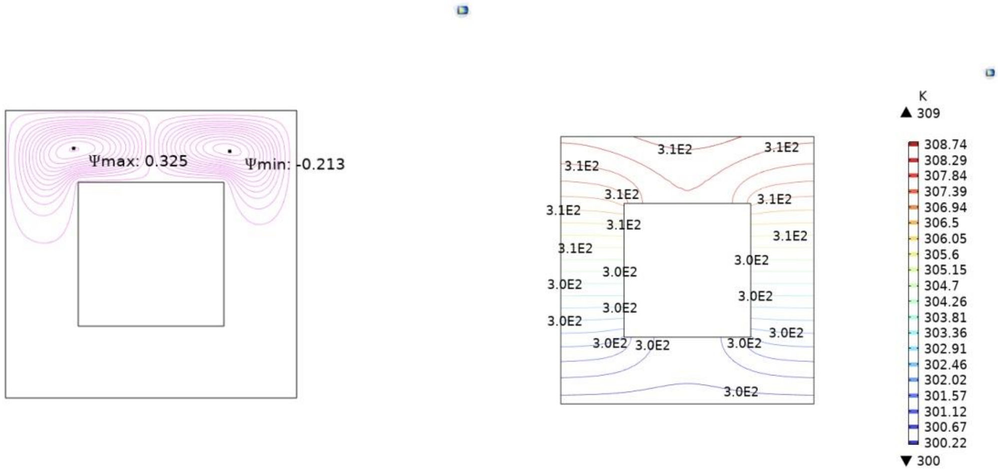


(a)Ra=10^3

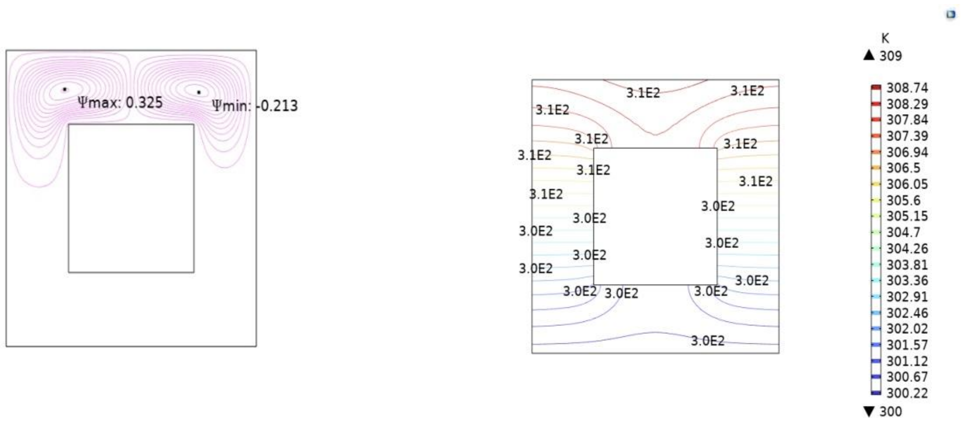


(b)Ra=10^7

FIGURE3: Isothermal lines and stream lines for case I (z=10, x=4) for (a)Ra=10^3 and (b)Ra=10^7



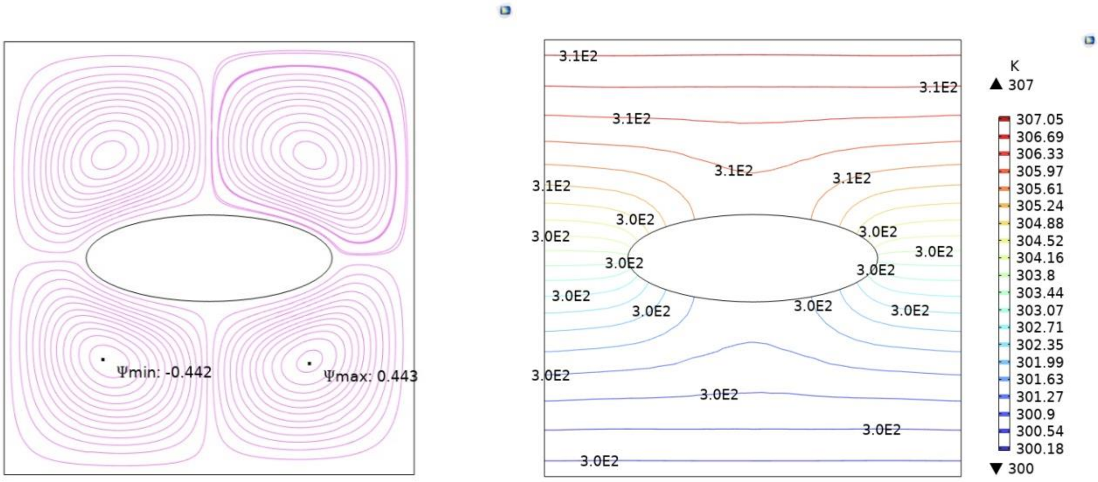
(a)



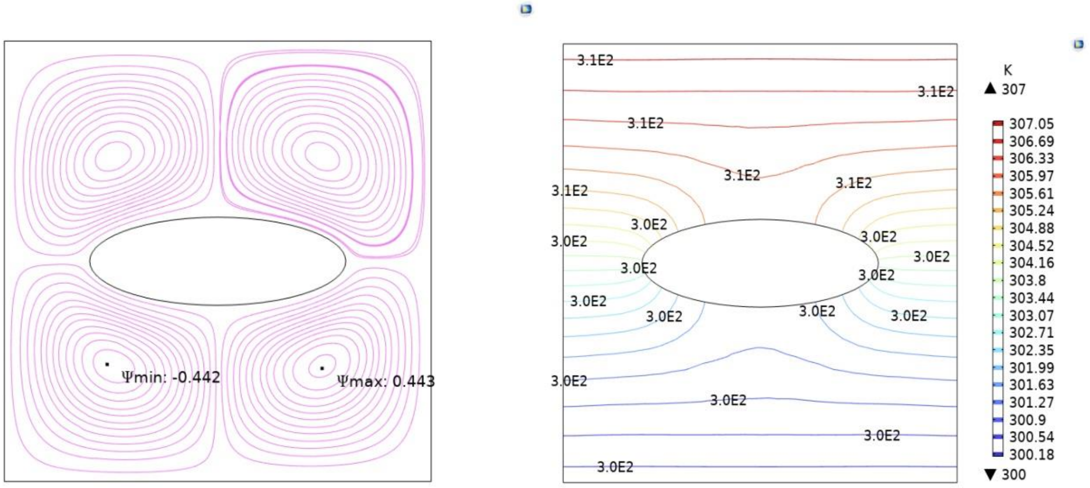
(b)

FIGURE4. Isothermal lines and stream lines for caseII (z=10,x=5) for (a)Ra=10^3 and (b)Ra=10^7

Figure4 shows isothermal lines as well as stream lines for case II when the side length of the enclosure is x=10 and the side length of the solid object is z=5. A huge difference in the computed results is observed in this case when it is compared to the first case. The purpose of the comparison is to check the influence of changing the geometrical shape of the inner solid object on the results. A huge difference in the intensity of the swirls is seen in this case for both values of Ra, this huge change in the distribution of the vortices of stream lines comes from changing the dimension of the inner object. This change has made a noticeable impact on the results. It is observed that the distribution of the swirls is concentrated only at the upper part of the enclosure. It is worthy to mention that the colors of the isothermal lines represents the gradation of the temperature starting from the blue at the lowest and ending with red at the highest. The accumulation of two main groups of vortices at the top is the main noticeable difference that was observed in this case , this reflects a lower quality of the convection currents which means that this case is note the perfect case for attaining the best results compared to the previous case. The comparison here is very important to attain the best distribution for stream line contours as well as isotherms. Furthermore, the highest and lowest values of the stream function Ψ is demonstrated on the figures of each case.



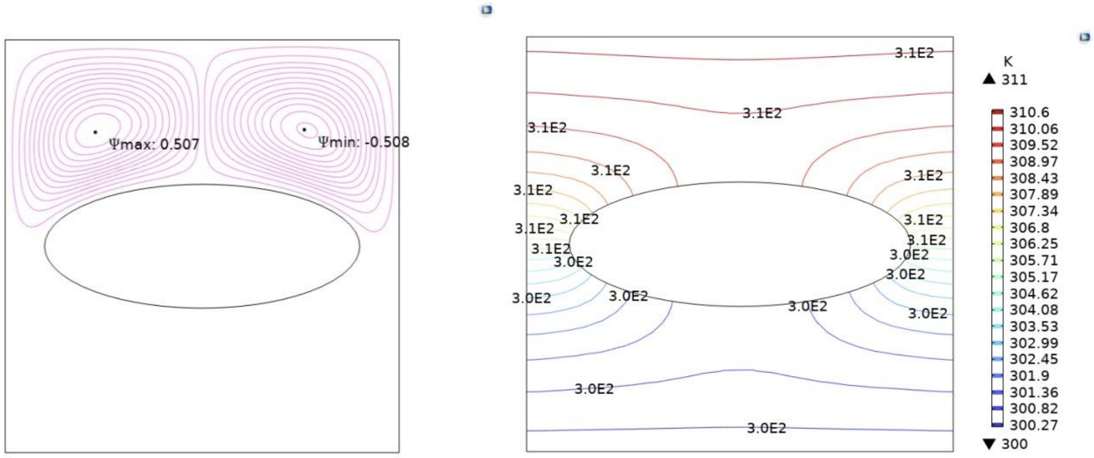
(a)



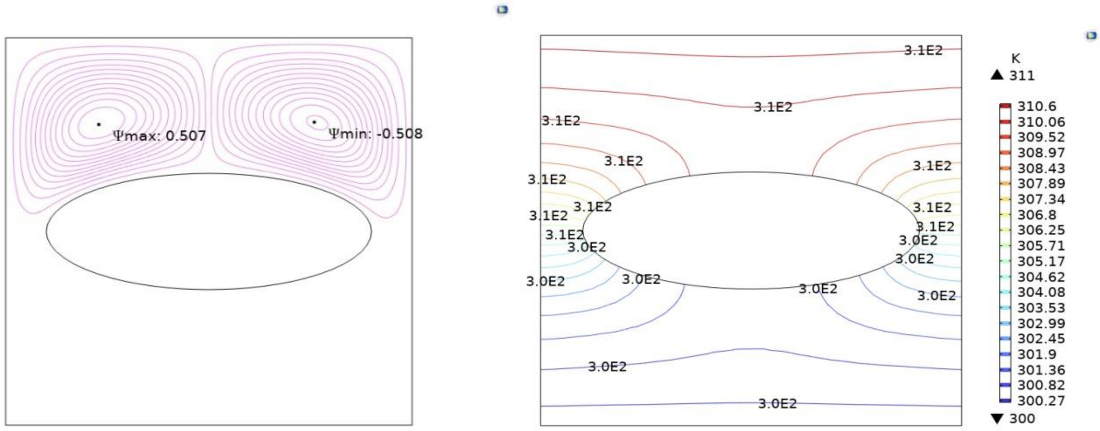
(b)

FIGURE5. Isothermal lines and stream lines for case III (z=10, a=3,b=1) for (a)Ra=10^3 and (b)Ra=10^7

Figure5shows the contours of the stream lines as well as isothermal lines demonstrated for the two different values of Ra. Changing the geometrical shape of the inserted inner solid object had an enormous impact on the computed results of the vortices of the convective flow and the distribution of them, also the location of the maximum and minimum values of the stream function has changed compared to the first and the second cases. It is observed here that the highest and lowest values of the stream function Ψ is located at the lower part of the enclosure unlike the first case. A large four groups of vortices is established to form the convective flow system , these four groups of swirls is distributed around the inserted oval inner solid object. A very good distribution of the isothermal lines is also observed in this case which gives a brilliant results of transmitting thermal energy through the system. The selected dimensions of the inner oval shape gave an excellent improvement on the attained results. A systematic distribution of the swirls and contours is seen and recognized in this case according to the selected geometrical shape of the solid object inserted inside the enclosure.



(a)



(b)

FIGURE6. Isothermal lines and stream lines for case IV (z=10, a=4,b=1.5) for (a)Ra=10^3 and (b)Ra=10^7

The demonstrated results in figure 6 shows the thermal flow system of case IV , where it is noticed that a two main group of vortices is formed and concentrated at the upper part of the enclosure while the distribution of the flow system is very poor at the lower part in this case. The values of the maximum and minimum value of the stream function is shown on the contours. On the other hand, the distribution of the isothermal lines is considered good where it starts on the sides of the oval shape also the highest intensity of these lines is observed on the right and the left sides while the intensity decreases at the upper and lower parts of the geometry.

Calculating the Ratio of Convective to Conductive Thermal Energy

The indicator that indicates the quality of the thermal flow system and specifies whether the flow is good enough to give the needed results by transmitting the desired amount of thermal energy is called Nusslet number. That's why it is very important to discuss the output of any computed thermal flow to check the efficiency of transmitting thermal energy. Nusslet number is computed for each case of different geometrical shape and the outputs are compared to recognize the best achieved results. It is important to mention that Nu is a dimensionless number. This figure gives an overall view to the values of Nu in each case and compares the results to check the level of thermal transmission.

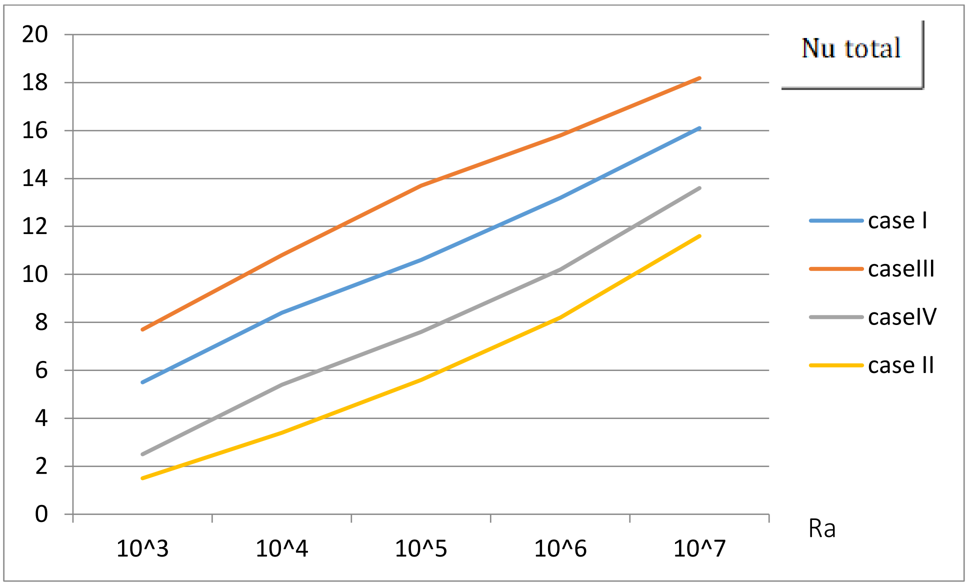


FIGURE7. Representation of the effect of different geometry on the outputs of Nu

It is obvious that the highest transmitted amount of thermal energy was achieved in case III which is shown on figure(7). By comparing the four studied cases, it was observed that the best result of transmitting thermal energy took place in case III represented by a Square enclosure of 10 cm side length contains an elliptic solid object of 3 cm a- semiaxis and 1cm b-semiaxis , while the second rank for the Nusslet number was achieved by case I represented by Square enclosure of 10 cm side length contains a square solid object of 4 cm side length . Last but not least, Case IV represented by a square enclosure of 10 cm side length involves an elliptic solid object of 4 a- semiaxis and 1.5 cm b-semiaxis achieved a range of Nusslet number starts from 2.3 and ends at 13.8. Finally, the least recorded values of Nu was recorded for case II , where we can see that the largest value of Nu in this case was less than 12. Consequently, it was recognized that the highest amount of thermal transmission took place in Case III which means Case III is the best case compared to the other cases. On the other hand, Case II achieved the least outputs of Nu number values , which means it has the lowest efficiency of transmitting thermal energy.

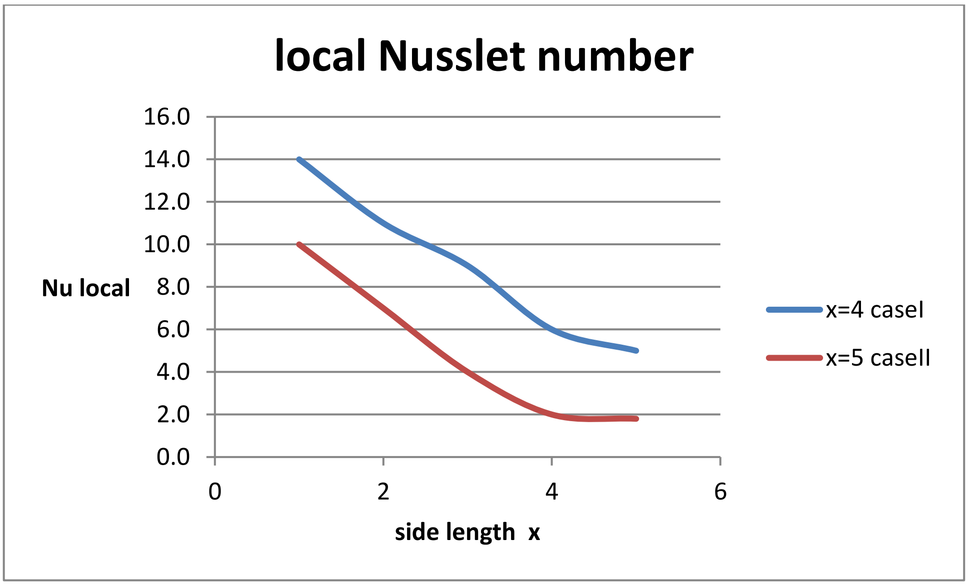


FIGURE8: comparsion between local Nu of case I and case II

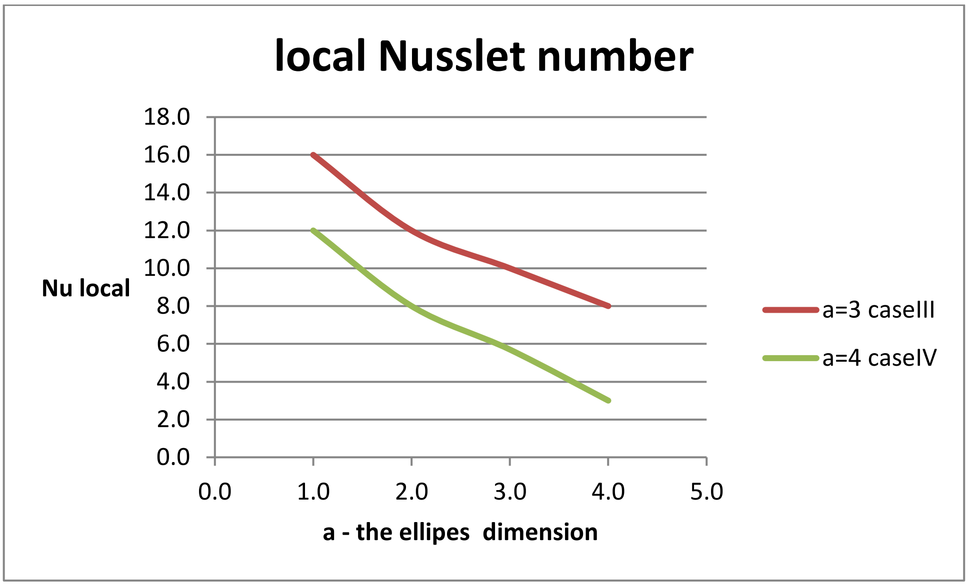


FIGURE9: comparison between local Nu of case III and case IV

Figure(8) and figure(9) shows the relationship between the variable dimensions of the geometrical shape of the inserted solid object and the values of local Nu number. In figure(8) there is two values of the side length of the inner solid square where (x=4) for case I while, (x=5) in caseII. The effect of increasing the side length of the inner solid object and consequently increasing the size of the inner solid object is clear in the figure where we can see a noticeable change in the values of the regional Nu. It is obvious that increasing the size of the inner object will lead to a decrease in the values of the local Nu according to the figure.

In figure(9) the relationship between (a) the dimension of the ellipse and the local Nu number is represented. For caseIII(a=3) and for caseIV(a=4). A remarkable difference in the values of the regional Nu number is recorded which clarifies that there is a huge impact of increasing the dimensions of the elliptic shape on lowering the outputs of the regional Nu. In other words, decreasing the dimensions of the inner elliptic solid object will lead to an increase in the values of the local Nu number.

5. CONCLUSION

The conjugate free convective thermal transmission through a square enclosure containing a solid object was analyzed numerically in this study. The results were presented in the form of stream lines and isothermal lines. The amount of transmitting convective to conductive thermal energy was computed in the form of total and local Nusslet number. Four cases were studied and the results of these cases were compared to check the best achieved outputs. The major principle of this study was to see the effect of changing the type of the geometrical shape and dimensions of the inserted solid object inside the enclosure. The following conclusions were observed and recorded according to the computed numerical results:

- ❖ Increasing the side length of the inner square solid object inside the square enclosure will lead to a decrease in the local Nusslet number.
- ❖ Increasing the dimensions of the elliptic solid object inside the square enclosure will lead to a decrease in the values of local Nusslet number.
- ❖ Decreasing the side length values of the inner square solid object inside the square enclosure had a large impact on the streamlines and isotherms.
- ❖ An excellent distribution of the streamlines and isothermal lines was achieved when lowering the side length values of the square solid object inside the square enclosure.

- ❖ Increasing the side length of the inner square solid object inside the square enclosure will give a poor distribution of the stream lines and isothermal lines and consequently a poor thermal transmission.
- ❖ Increasing the dimensions of the elliptic solid object inside the square enclosure will give a poor distribution of the stream lines and isothermal lines and consequently a poor thermal transmission.
- ❖ CaseI and CaseIII recorded the best distribution of isothermal lines and stream lines and consequently the highest values of the total Nu number which means the highest rate of thermal transmission compared to the other cases.

Nomenclature
x: side length of square solid object
z: side length of the square enclosure
a: major semi axis of the elliptic solid object
b: minor semi axis of the elliptic solid object
g: Gravitational acceleration
K_r: Thermal conductivity ratio
k: Thermal conductivity
ℓ: Width and height of enclosure
Nu_{loc}: local Nusslet Number
Nu_{avg}: Total Nusselt number
Q: heat flux
N: number of sides of the polygon
Ra: Rayleigh number
T: Temperature
Pr : Prandtle number
u, V: Velocity components in the x- and
y-directions
x, y&X, Y: Space coordinates & dimensionless space
coordinates.
TBL: Thermal Boundary Layer
Greek Symbols
α: Thermal diffusivity
β: Thermal expansion coefficient
Θ: Dimensionless temperature
v: Kinematic viscosity.
Subscript
c: Cold
f: Fluid

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